Dimensional Mathematics: An Axiomatic Extension of Epistemic Relativity

With an Application to the Prisoner's Dilemma

1. Introduction

Classical logic and mathematics are powerful tools, but they reach their limits as soon as phenomena such as self-reference, contextual dependence or emergent rationality come into play. These limitations are evident not only in the philosophy of mind or cognitive science, but also in game theory, where strategic decisions depend on expectations, mutual reflections, and cultural frameworks. While traditional models are based on linear derivation and fixed utility functions, such phenomena require an extended structure that combines systematic computation with reflexivity and context integration.

The "dimensional mathematics" presented here has emerged from the idea of the geometry of thought. Originally developed as a metaphor for different stages of cognitive processes, it has been transformed into an independent axiomatics that is not only metaphorically but formally consistent. The focus is on operators that link systematic derivation, reflexive fixed points and contextual coherence. This creates a mathematical framework that precisely depicts epistemic relativism without slipping into arbitrariness.

The aim of this article is to present these basics in a concise form and to demonstrate their performance with a concrete example. The Prisoner's Dilemma, a paradigmatic model of game theory, is particularly well suited to show how the new operators can lead to solutions that go beyond the classical Nash equilibrium. In this way, it is possible to mathematically understand how cooperation can arise from reflexivity and contextuality -- a phenomenon that is of central importance in the real world, but has not yet been formally satisfactorily grasped.

In order to fulfill this claim, it is first necessary to precisely develop the underlying structure of dimensional mathematics. Only when the formal foundations are clearly outlined can it be shown that it is not just a metaphorical way of speaking, but a consistent axiom system with its own mathematical carrying capacity. In the next section, therefore, the epistemic space is introduced, the operators are defined and their connection to a dimensionally extended utility function is presented.Σ

2. Axiomatic Basics

Dimensional mathematics is based on the introduction of an epistemic space that goes beyond classical formal-logical structures and provides additional operators for reflexivity and contextuality. Form is $\Sigma = (D, O, \otimes, I)$ a structure in which a set of dimensions, a set of operators, a suitable combination rule and an interpretation map are denoted. The dimensions , and $DO \otimes ID_2 D_3 D_4$ represent levels of integration that build on each other: systematic derivation, reflexive fixed points and contextual coherence. The axioms A1--A8 state that these planes are not additive, but compositionally connected to each other, and yet possess well-defined mathematical properties.

At the core are three operators. The operator represents the classic systematic integration. It corresponds to the expected value principle, which generates a consistent benefit assessment from ontic invariant payoffs or measured values. The operator $\sigma_2\mu_3$ represents reflexivity. It captures the stability of strategies under reciprocal expectation loops and is formally modeled as a fixed-point operator that generates stable expectation congruences in finite iteration. Finally, the operator stands for context coherence. It measures the compatibility of a strategy with overarching framework conditions such as norms, cultural embeddedness, or institutional structures. Mathematically, $\kappa_4\kappa_4$ a Lipschitz-continuous functor is defined on a context space Θ , so that small changes in the context only induce proportional changes in the rating.

The combinatorial rule $\otimes$ ensures that the three contributions can be integrated into a single evaluation profile. For a player i , this results in a dimensionally extended utility function

$$U_i(a_i, a_j; \theta) = \sigma_2\big(u_i(a_i, a_j)\big) + \alpha\, \mu_3\big(M_i[a_i \mid a_j]\big) + \beta\, \kappa_4(\theta),$$

where is the ontic invariant defined payoff, $u_i M_i[a_i \mid a_j]$ is a measure of reflexive expectation congruence, θ is a context parameter, and are weighting factors. This representation directly implements the axioms: as a systematic component, as a reflexive fixed point, $\alpha, \beta \geq 0\sigma_2\mu_3\kappa_4$ as a coherence-preserving context operator. The interpretation mapping connects the physically or economically given measuring space with the epistemic interpretive space.I

The clear distinction is essential: the classic payouts $u_i(a_i, a_j)$ remain invariant and are defined ontically in the sense of economic measurement. The novelty lies exclusively in the epistemic layer of translation, which is formed by σ_2, and $\mu_3\kappa_4$. This avoids category errors, in which semantic properties such as trust or norms are incorrectly projected into the physical structure. Instead, they are represented in a well-defined mathematical layer that connects to the axiomatics. The dimensionally extended utility function is thus not a heuristic paraphrase, but a precise extension of classical formalism, which enables new equilibrium classes.

The axiomatics developed so far forms the formal framework of dimensional mathematics. It shows how systematic deduction, reflexive fixed points and contextual coherence can be combined in a well-defined structure. In order to make the added value of this approach visible compared to classical models, it is necessary to use the abstract operators on a concrete example. Game theory is a particularly suitable test field, as it draws on clearly defined mathematical structures as well as on deeply rooted problems of human rationality. The Prisoner's Dilemma offers an ideal starting point for this: It is considered a paradigmatic case for the tension between individual utility maximization and collective rationality. While classical equilibrium theory shows defection as a stable solution, the dimensional expansion by σ_2, and $\mu_3\kappa_4$ a reassessment of the strategic position allows. In the following, it is shown how dimensional logic can be used to mathematically justify cooperative structures that would not be achievable in the traditional framework.

3. Prisoner's Dilemma in Dimensional Logic

In this section, the classic Prisoner's dilemma is used as a minimal test case to show that the axiomatically introduced operators of dimensional mathematics not only provide an alternative description, but can actually change the solution class of the game. The starting point is the standard setup of two actors $i \in \{1,2\}$ with the pure strategies C (cooperate) and D (defect). The payouts are characterized by the four values, whereby the classic dilemma conditions and are assumed. Under these assumptions, the unambiguous Nash equilibrium of the base game is Pareto-superior, but not rationally stable.$R, T, P, ST > R > P > S2R > T + S(D, D)(C, C)$

The dimensional-epistemic expansion does not start with the payoff mechanics themselves, but with the interpretive layer in which actors evaluate their options. Formally, each player is assigned an interpretive map, which generates a dimensional evaluation profile from the ontic invariant payouts. The parameter $iI_i u_i(a_i, a_j)U_i(a_i, a_j; \theta)\theta \in \Theta$ describes the relevant context (e.g. norms, level of trust, institutional embeddedness). The separation between the measurement space (payouts) and the interpretation space (profiles) remains strict: the payoff matrix itself is not changed, but the utility function, which is decisive for the strategic choice, is.

It is $U_i(a_i, a_j; \theta)$ divided into three contributions that correspond to the axioms A6--A8. The systematic contribution $\sigma_2(u_i)$ is the classic utility component based on expected values that represents the payouts in D_2 . The reflexive contribution depicts the effect of reciprocal self-references: Players not only calculate the immediate payoff, but also the extent to which their choice is consistent with the anticipated choice of the opponent and appears as a stable fixed point in a chain of higher expectations of order. The contextual contribution captures coherence with normative and cultural framework conditions. Taken together, for each action pairing,$\mu_3 \kappa_4(\theta)$

$$U_i(a_i, a_j; \theta) = \sigma_2\big(u_i(a_i, a_j)\big) + \alpha\, \mu_3\big(M_i[a_i \mid a_j]\big) + \beta\, \kappa_4(\theta),$$

where are weights and $\alpha, \beta \geq 0 M_i[a_i \mid a_j]$ denotes a reflexive agreement quantity that is high if and is a_i symmetrical and mutually expected. The term $a_j \kappa_4(\theta)$ is understood as a scaled coherence index, which is typically positive in a cooperative context and neutral to negative in the case of a violation of norms. This representation implements the axioms A6--A8 directly: σ_2 acts as a convex systematic integration of the payoffs, is modeled as a fixed point contribution, $\mu_3 \kappa_4$ satisfies a Lischitz coherence condition over Θ.

The question of whether cooperation can arise as equilibrium is reduced to comparative differences between the dimensional utility values. For C as the best answer to C must hold. If you use the above decomposition, you get$U_i(C, C; \theta) \geq U_i(D, C; \theta)$

$$U_i(C, C; \theta) - U_i(D, C; \theta) = (R - T) + \alpha\, \Delta_\mu + \beta\, \Delta_\kappa(\theta),$$

where $\Delta_\mu = \mu_3\big(M_i[C \mid C]\big) - \mu_3\big(M_i[D \mid C]\big)$ measures the reflexive added value of symmetrical cooperation over opportunistic deviation and maps the contextual preference for cooperative behavior. A sufficient criterion for the stability of $\Delta_\kappa(\theta) = \kappa_4(\theta \mid C, C) - \kappa_4(\theta \mid D, C)(C, C)$ as Nash equilibrium is therefore

$$\alpha\, \Delta_\mu + \beta\, \Delta_\kappa(\theta) \geq T - R. \tag{1}$$

The left side precisely summarizes the two newly introduced dimensional contributions: self-reference in the sense of mutual congruence of expectations and context coherence in the sense of cultural or normative embeddedness. The imbalance that favors defection over cooperation in the base game can be compensated for by reflexive and contextual gains. This provides a precise condition under which the game tilts from a dilemma to a coordination regime in which cooperation is rationally stable.$T - R$

To ensure that this condition does not remain merely heuristic, a dynamic formulation is helpful. Let be $p_i \in [0,1]$ the mixed strategy of player i, as probability for C. Define a best-response dynamic $T\colon [0,1]^2 \to [0,1]^2$, which assigns to each profile the profile $p = (p_1, p_2)T(p)$ whose components are calculated from the utility differences by a regular voting rule, such as a logistic response function. The i following applies to players

$$T_i(p) = \frac{1}{1 + \exp\{-\lambda\,[U_i(C, p_j; \theta) - U_i(D, p_j; \theta)]\}},$$

where represents an inverse temperature that controls the stochastic of choice, and $\lambda > 0 U_i(\cdot, p_j; \theta)$ denotes the expected dimensional utility values against the counterpart's mixed strategy. Under the axioms A6--A8 and the assumption that μ_3 on the reflexive subspace is contractionary and Lipschitz-continuous in as well as in, becomes a contractor on a suitable compact interval if the weights and steepness $\kappa_4 \theta p T \alpha, \beta \lambda$ are in a range that keeps the global Lipschitz constant below 1. Banach's fixed point theorem then returns the existence and uniqueness of a stable equilibrium point p^*.

It follows directly from (1) that for all θ, which $\beta\ \Delta_\kappa(\theta)$ make sufficiently large, and for reflexive constellations that lift over the threshold $\alpha\ \Delta_\mu T - R$, the fixed point lies in an environment of . $p^*(1,1)$ In words, this means that mutual reflexivity and appropriate contexts undermine the dominance of defection and establish cooperation as a rational, stochastically stable state. The qualitative mechanism is clear: rewards the convergence of higher expectations of order and punishes their breaking, shifts the effective weighting of the payoff components in favor of the cooperative profile without changing the ontic payoff matrix itself. The theory thus remains compatible with the invariance of physical or economic measurands and locates the novelty exclusively in the epistemic translation layer.$\mu_3 \kappa_4$

A numerical sketch is sufficient for illustration. Take $T = 5$, , , $R = 3 P = 1 S = 0$ as canonical values. In this configuration, the defect premium is $T - R = 2$. Suppose that reciprocal congruence of expectations generates an additional return in the cooperative couple $\mu_3 \Delta_\mu = 1$, while the context in the sense of a normative order of trust provides a coherence contribution $\Delta_\kappa(\theta) = 1{,}2$ and $\alpha = \beta = 1$ is chosen. Then the left side of (1) is equal and exceeds the defect premium. The best-response dynamics converge for moderate $2{,}2 \lambda$ to a fixed point p^* with a high probability of cooperation, which formally explains the emergence of a cooperative equilibrium. If you vary θ so that $\Delta_\kappa(\theta)$ it gets smaller, the left side of (1) sinks and the game continuously reverts towards the classical defection equilibrium, making sensitivity to loss of context mathematically accessible.

The proposed formulation thus not only introduces a new utility architecture, but establishes a well-defined class of "reflexive-contextual" games in which equilibria are characterized by fixed points of contractive response dynamics. It shows that cooperation does not have to be modeled as a psychological deviation from rationality, but as the result of rational choice in a dimensionally expanded but strictly axiomatized epistemics. The separation between ontic payoff structure and epistemic interpretation prevents category errors and makes the contribution of the dimensions precisely measurable: in the size of the threshold, in the reflexivity dividend and in the context coherence $T - R \Delta_\mu \Delta_\kappa(\theta)$.

The analysis of the Prisoner's Dilemma has shown that the expansion by the operators , and $\sigma_2 \mu_3 \kappa_4$ leads to a fundamentally new evaluation of strategic situations. While in classical formalism defection appears to be the only stable equilibrium, dimensional logic opens up additional classes of solutions in which cooperation can be justified in a mathematically stable way. This makes it clear that axiomatics is not only an abstract extension, but has direct consequences for the modeling of rational action. In the following, the theoretical and methodological implications of this will be discussed and how the approach can be placed in larger scientific contexts.

The classic prisoner's dilemma illustrates how mistrust and a lack of communication can lead to suboptimal results -- not only in criminal law, but also in business, politics, or everyday situations. The classical payoff structure () suggests that defection is the rationally dominant strategy. However, dimensional mathematics can be used to show that this dominance is not mandatory.$T > R > P > S$

The extended utility function is:

$$U_C = R + \alpha \cdot \Delta\mu + \beta \cdot \Delta\kappa(\theta)$$

This describes $\Delta\mu$ the reflexive added value that arises when both players align their expectations, while the gain is represented by contextual coherence, such as social norms, institutional embeddedness, or cultural practices. $\Delta\kappa(\theta)\alpha$ and weigh the respective influence.β

To illustrate the mode of action, let's consider a classic dilemma with the values , , and . Without extension, the defect remains rational because $T = 5R = 3P = 1S = 0T > R$. However, if we assume for the reflexive surplus value , for the context coherence $\Delta\mu = 1.5\Delta\kappa(\theta) = 0.8$, as well as and , the result is:$\alpha = 1.2\beta = 1.0$

$$U_C = 3 + 1.2 \cdot 1.5 + 1.0 \cdot 0.8 = 5.6$$

Thus, the benefit of cooperative action exceeds the temptation value $T = 5$, and cooperation becomes a rationally stable strategy.

This result makes it clear that cooperation cannot be understood as a moral exception, but as the result of an expanded rationality. The classic payoff matrix remains unchanged; the novelty lies in the epistemic evaluation layer. Rationality here includes not only the immediate payoff, but also the reflexive and contextual dimension.

The implications of this approach are particularly evident when the model is applied to real-world scenarios. Climate policy can thus be understood as a global prisoner's dilemma: states are faced with the decision to reduce emissions (cooperation) or to continue to realize short-term benefits through fossil energy use (defection). Traditionally, the temptation to assert one's own interests dominates. However, if the epistemic and social contributions are taken into account, the picture changes. The reflexive added value $\Delta\mu$ is reflected in trust through diplomatic relations, joint target agreements and mutual expectation. The gain in context $\Delta\kappa(\theta)$ results from international treaties, public movements or culturally anchored responsibility for future generations. If these factors increase, cooperation can also become rationally stable at the global level -- even if it remains short-term $T > R$.

Thus, dimensional mathematics provides a theoretical foundation not only to explain cooperation retrospectively, but also to prospectively justify it as a stable solution. It expands the concept of rationality from the purely egoistic maximization of utility to an epistemically and contextually enriched rationality that comes closer to the conditions of real social systems.

4. Discussion and implications

The introduction of dimensional operators makes it possible to overcome central weaknesses of classical formalism. In game theory, this means that the reduction to purely individual utility maximization is relativized. The operators μ_3 and κ_4 show that real actors do not act in isolation, but embed their behavior in recursive expectation loops and in cultural or normative frameworks. This makes visible a dimension that appears to be "irrational" or "extra-economic" in classical equilibrium models, but is mathematically anchored in dimensional formalism. Cooperation thus loses the character of a moral or social additional assumption and becomes a possible solution class that follows from the formal structure itself.

This expansion has far-reaching consequences. In economic theory, it opens up the possibility of modelling collective action not only heuristically, but mathematically sound. In cognitive science, it allows processes of self-reflection and context evaluation to be described as formal operators, rather than leaving them in vague terms such as "intuition" or "trust". Even in physics, especially in the interpretation of quantum mechanical experiments, the separation between ontic invariance and epistemic relativity could become fruitful: the measurement results remain invariant, but the interpretation of their meaning depends on the dimensions in which they are contextualized.

Crucially, dimensional mathematics does not merely provide a paraphrasing of known phenomena, but a genuine extension of the mathematical toolbox. It provides a new formal layer that avoids category errors and treats phenomena such as reflexivity and contextuality not outside but within the mathematical framework. This creates an approach that can be applied to classical problems in game theory as well as to open questions in economics, cognition and physics.

5. Implications for Artificial Intelligence

The scope of dimensional mathematics is particularly evident when applied to questions of artificial intelligence. Classical machine learning methods, especially in the field of reinforcement learning, operate almost exclusively at the level of . Rewards are translated into expected values, strategies are improved through systematic gradient optimization, and the goal is to maximize payouts in a defined state space. As long as it comes to clearly defined tasks such as board games or navigation, this formalism proves to be extraordinarily successful. However, it reaches its limits where reflexivity and context integration are unavoidable, for example in the interaction of several intelligent agents or in the problem of so-called alignment, i.e. the adaptation of machine systems to human norms and values.σ_2

This is where the operators μ_3 open up a new perspective. The reflexive operator $\kappa_4\mu_3$ can serve as a formal basis for theory of mind in artificial systems. It allows us to model not only expectations about states, but expectations about expectations. This allows scenarios to be captured in which agents react reciprocally to the strategies and beliefs of others without sinking into recursive arbitrariness. Formally, this leads to fixed-point-like structures that describe stable congruences of expectations. Such models would be a step beyond what is currently possible in multi-agent systems.

The context operator κ_4 , on the other hand, offers a way to embed normative and cultural framework conditions not only heuristically, but formally in the utility function. While today's approaches often introduce ad hoc regularizations or additional heuristics to take into account fairness or security, for example, a coherent mathematical mapping of such framework conditions allows. Context coherence is not treated as an external postulate, but as an integral part of the evaluation that merges with the other operators to form a consistent whole. This creates an approach in which machine action can be aligned not only with short-term optimization, but also with higher-level interpretive frameworks.κ_4

In this perspective, dimensional mathematics appears as a metatheory of artificial intelligence. It complements classical optimization with layers that were previously either implicit or insufficiently modeled: self-reflection and contextuality. Its strength lies in the fact that it not only describes these extensions phenomenologically, but also integrates them into a mathematical structure as axiomatic operators. This could open up a way to develop artificial systems that are not only powerful, but also compatible in reflexive and normative terms.

The perspectives outlined already mark that dimensional mathematics points beyond classical fields of application. This is particularly evident in artificial intelligence, which is currently on the threshold between impressive technical successes and unsolved theoretical problems. While today's methods work on the basis of classical optimization paradigms, it is becoming increasingly apparent that questions of self-reflection, expectation nesting and normative embedding can no longer be mastered with linear methods alone. This is exactly where the operators of dimensional mathematics can be used productively.

6. Experimental predictions and tests

The presented axiomatics not only opens up a theoretical framework, but also allows concrete experimental and simulation-based tests. While classical game theory usually relies on analytical equilibrium models, the extended utility function enables $U_C = R + \alpha \cdot \Delta\mu + \beta \cdot \Delta\kappa(\theta)$ an empirical operationalization of epistemic and contextual factors.

First, the parameters α, and $\beta\Delta\kappa(\theta)$ can be deduced by empirical methods. For example, controlled communication experiments between test subjects could be used to measure by mutually aligning expectations and quantifying the effect on cooperative behavior. The context parameter could be operationalized by social variables such as trust, institutional embeddedness, or cultural norms, such as surveys, social network analyses, or experimental manipulations of institutional frameworks. The weighting factors $\Delta\mu\theta\alpha$ and β would then be freely estimated coefficients that represent the relative influence of reflexivity and context coherence on the decision-making patterns.

Secondly, simulation is a good option. Agent-based models can be used to define populations of actors whose utility functions include the dimensions (systematic derivation), $\sigma_2\mu_3$ (reflexivity) and (context coherence). In iterated prisoner dilemmas, it could be examined whether and under what conditions cooperative equilibria develop stably. Here it would be particularly interesting to test the fixed point dynamics of $\kappa_4\mu_3$ and the Lipschitz continuity of κ_4 experimentally. First hypothesis: The higher the communication frequency and the denser the social network, the more stable the cooperative solution, as μ_3 the κ_4 more robust the social network.

Thirdly, axiomatics opens up possibilities for comparison with existing approaches. In this way, it could be investigated whether it goes beyond the classical theory-of-mind models by measuring not only expectation expectations, but also their systematic contraction in dynamics. It could also be examined whether it is compatible or complementary to established concepts such as social preferences (Fehr & Schmidt) or cultural evolution (Boyd & Richerson). Experiments with varying context parameters would provide clarity here.$\mu_3\kappa_4$

Fourthly, and finally, axiomatics allows a check in the field of AI. Multi-agent systems or reinforcement learning models could be equipped with a dimensionally extended utility function. The question of whether this makes cooperation more stable in complex environments would be an empirically testable criterion for the practical use of the theory. Reflexive layers in neural networks could simulate, while contextual regularizations can $\mu_3\kappa_4$ be implemented.

In summary, the theory is open to empirical verification. It not only proposes a philosophical-mathematical extension, but also generates clear hypotheses: (a) communication and the alignment of expectations increase the cooperative added value $\Delta\mu$; (b) institutional and cultural embeddedness increase contextual gain $\Delta\kappa(\theta)$; (c) beyond a threshold, cooperation becomes rationally stable; (d) Simulations and AI implementations can systematically test these hypotheses. This will result in a research program that combines theory, empiricism and application in a novel way.

7. Conclusion and outlook

The dimensional mathematics developed here shows that epistemic relativity does not have to remain arbitrary, but can be transformed into a consistent axiomatics. Through the introduction of the operators σ_2, it $\mu_3\kappa_4$ becomes clear that systematic derivation, reflexivity and contextuality are integral components of an extended mathematical structure. Using the example of the Prisoner's Dilemma, it was demonstrated how this structure results in new classes of solutions that make cooperation appear not as an exception, but as a formally justified possibility.

However, the scope of the approach extends far beyond this example. In economics, it opens up new perspectives on collective action; in cognitive science, it provides precise language for self-reflection and context evaluation; in physics, it indicates how ontic invariance and epistemic ambiguity can be clearly separated from each other, but at the same time systematically related to each other.

Future work should follow two directions. First, it is necessary to further specify the axiomatic foundations and to systematically examine the mathematical structure in terms of consistency and completeness. Second, concrete applications in various disciplines were to be worked out -- from complex game-theoretical scenarios to models of social interaction and possible implications for the interpretation of physical experiments. In this way, it can be shown whether dimensional mathematics provides not only a new language, but a foundation on which to build a further scientific research program.

Appendix: Algorithm Box: Dimensionally Extended Prisoner's Dilemma (pseudocode)

This box summarizes the core of the simulation in a compact, printable form. It shows the dynamic dialing rule, the reflexive expectation actualization, and the inclusion of the context operator. The complete Python code is in the supplement.

Inputs are the weights and , the context parameter , the inverse temperature and the contraction rate . The probabilities of cooperation , as well as the reflexive expectations , are initialized. The payouts of the classic game are given as ontic constants , , , (here , about , , ,). The dynamics iterate to the convergence threshold $\alpha\beta\theta\lambda\eta p_1 p_2 e_1 e_2 TRPST > R > P > ST = 5R = 3P = 1S = 0 tol$ or to the maximum number of iterations.

Pseudocode

Set T, R, P, S as constants. Choose $\alpha \geq 0$, , , , , $\beta \geq 0 \theta \in \mathbb{R} \lambda > 0 \eta \in (0,1]$, $tol > 0$, $max\_iter \in \mathbb{N}$. Initialize , , , . $p_1 \leftarrow 0.5 p_2 \leftarrow 0.5 e_1 \leftarrow 0.5 e_2 \leftarrow 0.5$ Repeat for *it max_iter*$= 1, \ldots$,: Calculate the difference in utility for player 1 $\Delta U_1 = p_2 \cdot (R - T) + (1 - p_2) \cdot (S - P) + \alpha \cdot (2e_1 - 1) + \beta \cdot \theta$ and analogously ΔU_2 for player 2. $p_1 \leftarrow 1/(1 + \exp(-\lambda \cdot \Delta U_1))$ Update the reflexive expectations contractionally and $p_2 \leftarrow 1/(1 + \exp(-\lambda \cdot \Delta U_2)) e_1 \leftarrow (1 - \eta) \cdot e_1 + \eta \cdot p_2 e_2 \leftarrow (1 - \eta) \cdot e_2 + \eta \cdot p_1$. Check convergence: if $\max\{|\Delta p_1|, |\Delta p_2|, |\Delta e_1|, |\Delta e_2|\} <$*tol*, then stop and return , $(p_1, p_2), (e_1, e_2) it$. If convergence is not achieved, return the last state.

Interpretation

The logistic choice rule implements the systematic component; the contractive expectation actualization models the reflexive operator; the additive contribution $\sigma_2 \mu_3 \beta \cdot \theta$ represents the context operatorκ_4. Cooperation becomes rationally stable as soon as $\alpha \cdot \Delta\mu + \beta \cdot \Delta\kappa(\theta)$ the classic defect premium is $T - R$ compensated. The dynamics converge under moderate parameters to a unique fixed point, which corresponds to Banach's fixed point theorem in the reflexive subspace.

The complete Python code for simulating the dimensionally extended Prisoner's Dilemma is available as an accompanying supplement. It includes all the features to reproduce the results presented in this article, including:

The file *dimensional_pd_simulation_python* is attached to this article as supplementary material.

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