Dimensional Logic as a Formal Inference System

An Extension of Predicate Logic for Epistemically Layered Thinking

Abstract

Classical predicate logic operates under the assumption of dimensional homogeneity: statements are evaluated as either true or false, regardless of their epistemic context or the cognitive level at which they operate. This article develops a systematic extension of symbolic logic that removes this limitation. Dimensional logic introduces explicit epistemic dimensions into formal syntax, relativizes the truth predicate to multidimensional profiles, and establishes transition rules between different levels of knowledge. The resulting system retains the advantages of classical logic within the systematic dimension, while formally integrating immediate perception, reflexive metacognition, and contextual embedding. We use detailed examples (prisoner's dilemma with concrete parameters, ethical decisions, scientific statements) to show that this extended system can formally distinguish categorically different modes of validity and thus make category errors detectable as formal violations.

Keywords: Dimensional logic, epistemic dimensions, predicate logic, transition operators, reflexive metacognition, contextual embedding

1. Introduction: The Problem of Dimensional Homogeneity

Classical logic, as developed since Frege (1879) and Russell (1910) and semantically substantiated by Tarski (1933/1956), is based on the principle of bivalence: a statement is either true or false, tertium non datur. This structure has the undeniable advantage of complete formalizability and allows for a clear, deductive system (Hodges, 2013). In combination with set theory, modal logic and predicate logic, a system was created that can uniformly grasp formal proofs, mathematical definitions and algorithmic structures.

But this system is based on a fundamental abstraction: that truth can be determined independently of the subject, the context and the level of cognitive processing. This assumption proves to be legitimate in many applications, especially in arithmetic, geometry and algorithmic structuring. However, as soon as it comes to meaning, interpretation, social reality or subjective experience, binary validity loses its epistemological ground (Putnam, 1981).

The well-known paradoxes of self-referentiality, such as liars or Gödel's incompleteness theorems (Gödel, 1931), already show that there are statements that elude a clear true-false assignment without being meaningless. Even in mathematics itself, statements such as the axiom of choice or the continuum hypothesis are formally undecidable, their validity depends on the axiomatic system chosen (Cohen, 1963). This insight is not new.

Since the 20th century, alternative logic systems have developed: multivalued logics (Łukasiewicz, 1920), intuitionistic logic (Brouwer, 1923), modal (Kripke, 1963), temporal and paraconsistent logics (Priest, 2006). But all these systems remain within a formal-structural framework, they modify values of validity or operators, but not the mode of thinking itself.

What is missing is an approach that epistemologically starts from the structure of thought. Human thinking is not flat, but layered (Piaget, 1952; Tomasello, 2019). It moves through stages of distinction, structuring, reflexive self-referentiality and contextual embeddedness. These stages can be described as epistemic dimensions, and each of these dimensions creates its own type of validity. Dimensional logic, as developed here, takes this layering seriously and integrates it into the formal structure of logic itself.

1.1 The Four Epistemic Dimensions

Dimensional logic distinguishes four basic epistemic modes, which build on each other and each constitute a specific space of validity (Stegemann, 2025a).

The first dimension, D_1, captures immediate perception of difference and pre-linguistic distinction. Here, differences are felt, perceived, experienced. A child who distinguishes between warm and cold, light and dark, closeness and distance moves on this level (Piaget, 1952). Validity in means: something is experienced as different, not logically derived, but given phenomenally. This dimension forms the basis of all further abstraction.D_1

The second dimension, , is the level of classical logic and formal structure. This is where thinking in the narrower sense begins: distinctions become concepts, concepts become statements, statements become derivations. Classical two-valued logic operates on this level with truth values, operators and axioms (Frege, 1879; Russell, 1910). is the level of explicit, rule-guided structuring, its validity results from inner consistency and deductive consistency. Statements such as or are anchored in this dimension.

$$D_2 D_2 2 + 2 = 4 \forall x: P(x) \rightarrow P(x)$$

The third dimension, D_3, adds reflexive metacognition. Here, not only systematic thinking is done, but thinking itself becomes the object. This includes the ability to change perspectives (theory of mind), to model mutual expectations and to meta-theoretical reflection on one's own cognitive processes (Premack & Woodruff, 1978; Tomasello, 2019). This dimension includes, for example, the ability to recognize that the other person perceives the situation differently, or the insight that one's own beliefs have prerequisites that can be questioned. The reflexive dimension allows for recursive thought structures: I think about what you think about what I think.

The fourth dimension, D_4, integrates contextual and paradigmatic embeddedness. Here it is no longer just a matter of reflecting on one's own thinking, but of embedding all cognitive processes in historical, cultural and institutional frameworks (Kuhn, 1962;

Foucault, 1969). Validity is understood here as relative to paradigms, traditions, normative orders. Scientific theories operate at this level when they make their own paradigmatic assumptions transparent: Newtonian mechanics applies in the context of low speeds, quantum mechanics in the context of microscopic phenomena. It D_4 becomes apparent that truth is not only reflexive, but also historically and culturally situated.

These four dimensions are not chosen arbitrarily, but result from an analysis of cognitive development itself. They correspond to Piaget's developmental stages (Piaget, 1952): D_1 corresponds to the sensorimotor phase, D_2 the concrete and formal-operational stages, while D_3 D_4 the sensorimotor phase and the concrete and formal-operational stages are extensions that go beyond Piaget and capture the reflexive and socio-historical dimension of cognitive maturity.

1.2 Why a new logic?

The question that immediately arises is: If classical logic works perfectly within , D_2 why extend it? The answer lies in the recognition that many philosophical and scientific problems result from unrecognized dimensional transitions (Ryle, 1949). If a statement that applies to is D_2 transferred to or transferred without reflection $D_3 D_4$, or vice versa, if reflexive or contextual validity is misinterpreted as universal truth, systematic category errors arise.

For example, the statement "electrons have spin" operates $\frac{1}{2}$formally and systematically within quantum mechanics. Viewed on this basis, it requires reflexive insight into the construction processes of quantum theory and the reciprocal relationship between theory and experiment. It is understood as embedded in the paradigm of quantum physics, with all its historical and cultural prerequisites (Heisenberg, 1958). These questions cannot be formulated within classical logic because they address the epistemic status of the statement, not just its content.$D_2 D_3 D_4$

Dimensional logic is thus not a departure from classical logic, but its epistemological situation. It makes explicit what remains implicit in classical treatment: the epistemic level on which a statement operates and the conditions of its transition between different levels.

2. The Formal Language of Dimensional Logic

In order to formally grasp the dimensional structure of thought, we extend the standard first-level predicate logic with explicit dimension markers. This is not done by modifying the truth values themselves, but by typifying the terms and formulas.

2.1 Syntax

The basic building blocks of language remain unchanged: we have variables $x, y, z, \ldots$, constants $a, b, c, \ldots$, fixed-digit predicate symbols $P, Q, R, \ldots$, and function symbols

(Church, 1956). The crucial difference lies in the dimensional annotation of each formula.$f, g, h, \ldots$

A dimensionally typed atomic formula has the form:

$$P(t_1, \ldots, t_n)^{(d_i)}$$

where $d_i \in \{D_1, D_2, D_3, D_4\}$ indicates the epistemic dimension in which the statement operates. For the higher dimensions D_3 and D_4 additional parameters are introduced:

$$P(t_1, \ldots, t_n)^{(D_3,R)}$$

denotes a statement with a reflexive structure R (such as mutual expectations or metatheoretical conditions), while

$$P(t_1, \ldots, t_n)^{(D_4,C)}$$

marks a statement in context (such as a cultural paradigm or institutional order).C

The logical operators are also annotated dimensionally. A conjunction within the dimension is written as:d

$$\varphi^{(d)} \wedge^{(d)} \psi^{(d)}$$

Analogous to disjunction, negation, and implication. This ensures that logical joins remain within the same dimension as long as no explicit transformation is performed.

Complete notation of a statement includes not only the dimension but also a degree of validity:

$$\varphi^{(d,\alpha,\beta)}$$

where indicates the dimension of thought, $d\alpha$ additional parameters (reflexive structure or context) and the degree of validity. The latter allows fuzzy-logical extension (Zadeh, 1965) if necessary, but is optional for the basic structure.$\beta \in [0,1]$

2.2 Transition Operators

The core of dimensional logic lies in the operators that formalize transitions between dimensions. These operators are not classical logical operators, but meta-operators that transform the epistemic level of a statement.

The Up Operator $\uparrow (d_i \to d_j)$ transforms a statement from a lower dimension d_i to a higher dimension d_j (using):$i < j$

$$\uparrow (d_i \to d_j) \colon \varphi^{(d_i)} \mapsto \varphi^{(d_j)}$$

This transformation adds epistemic structure. For example, a formal-systematic statement raises to the reflexive level by embedding it in metatheoretical considerations.$\uparrow (D_2 \rightarrow D_3)$

The down operator projects a statement from a higher dimension to a lower one (with $\downarrow (d_j \rightarrow d_i) j > i$):

$$\downarrow (d_j \rightarrow d_i): \varphi^{(d_j)} \mapsto \pi(\varphi)^{(d_i)}$$

There is a projection function that reduces or abstracts information. For example, removes $\pi \downarrow (D_4 \rightarrow D_2)$ the context from a contextual statement and retains only the formal-systematic structure.

The integration operator $\bigoplus^{(d_k)}$ links statements from different dimensions to form an integrated statement in a common target dimension:

$$\bigoplus^{(d_k)}: \varphi^{(d_i)} \times \psi^{(d_j)} \mapsto \chi^{(d_k)}$$

where typically $k = \max(i, j)$ holds because the integration takes place in the highest dimension involved.

The convergence operator $\int^{(d)}$ aggregates multiple statements within the same dimension into a coherent overall statement:

$$\int^{(d)}: \{\varphi_1^{(d)}, \varphi_2^{(d)}, \ldots\} \mapsto \Phi^{(d)}$$

This operator is particularly relevant for the integration of empirical data or the synthesis of different theoretical perspectives within a common framework.

3. Semantics: Dimensionally layered models

Tarski semantics for classical predicate logic defines truth relative to a structure $\mathcal{M} = (D, I)$, where D is the domain and an interpretation function that maps predicates to sets of tuples (Tarski, 1933/1956). Dimensional logic extends this approach to a layered semantics, in which each dimension has its own interpretive structure.I

3.1 Definition of Dimensional Models

A dimensional model consists of:$\mathcal{M}$

$$\mathcal{M} = (D_1, D_2, D_3, D_4, I_1, I_2, I_3, I_4, T)$$

where for each dimension $i \in \{1,2,3,4\}$:

- D_i is the domain of the dimension, i.e. the set of objects that are quantified in this dimension.i

- I_i is the interpretation function for dimension that assigns meanings to predicates and function symbols in that dimension.i
- T is the set of allowable transition functions between dimensions.

The domains can be identical () or different, depending on whether the dimensional structure is an ontological or purely epistemic stratification. In the version proposed here, the ontology is monistic (a common domain), but the epistemic modes are plural (different interpretation functions).$D_1 = D_2 = D_3 = D_4$

3.2 Truth in Dimensional Models

Truth is dimensionally relativized. For a statement, $\varphi^{(d_i)}$ we define:

$$\mathcal{M} \vDash \varphi^{(D_1)} \iff \varphi \text{ is immediately perceptible/experienced in } \mathcal{M}$$

There D_1 is no propositional structure in the classical sense. Truth means phenomenal presence or experienced difference (Husserl, 1913).

$$\mathcal{M} \vDash \varphi^{(D_2)} \iff I_2(\varphi) = \text{true}$$

In , the classic Tarski semantics applies. A statement is true if it is fulfilled under the interpretation $D_2 I_2$. This is the standard definition used in classical logic (Tarski, 1933/1956).

It is important to emphasize that Tarski's semantic theory of truth already has a reflexive structure: it operates with the distinction between object language and metalanguage, where truth for object language is defined in a richer metalanguage (Tarski, 1933/1956). This shows that classical logic is not completely flat either, but has a hierarchical structure. However, Tarski's hierarchy remains purely formal-systematic: both object and metalanguage operate in the mode of systematic formalization (D_2). What is missing is, firstly, the integration of reflexive metacognition (D_3), which goes beyond the formal hierarchy and includes mutual expectations as well as self-referential thought processes, secondly, the embedding in paradigmatic and cultural contexts (), thirdly, the explicit typification of each formula according to its epistemic dimension, and fourthly, the formal transition operators that regulate transitions between epistemic modes. Dimensional logic thus expands Tarski's meta-level with additional epistemic structures that go beyond the formal-hierarchical separation.D_4

$$\mathcal{M} \vDash \varphi^{(D_3,R)} \iff I_3(\varphi \mid R) = \text{true}$$

In it, truth is structured reflexively. is a reflexive structure that specifies metatheoretical conditions, mutual expectations or changes of perspective. The interpretation does not evaluate absolutely, but relative to the reflexive structure.$D_3 R I_3 \varphi$

$$\mathcal{M} \vDash \varphi^{(D_4,C)} \iff I_4(\varphi \mid C) = \text{true} \wedge C \text{ ist paradigmatic consistent}$$

In , the contextual embedding itself becomes part of the evaluation. D_4C contains paradigmatic, cultural or institutional framework conditions. The truth of a statement on D_4 requires not only that it be valid under, but also that it itself is historically and culturally consistent.CC

3.3 Dimensional Truth Profiles

A central innovation of dimensional semantics is the concept of the truth profile. Instead of simply classifying a statement as true or false, it is given a four-dimensional profile:

$$V(\varphi) = (v_1, v_2, v_3, v_4) \text{ mit } v_i \in [0,1]$$

where indicates the degree of validity in dimension $v_i D_i$. This allows for a differentiated analysis: A scientific statement such as "light is a wave" could have the profile $V = (0.3,1.0,0.7,0.8)$, which means that it applies to (formal-systematic) full, to $D_2 D_3$ (reflexively taking into account metatheoretical considerations) restricted, to D_4 (contextually in the classical paradigm) strongly applies, and D_1 is hardly accessible to (immediate perception).

4. Axiomatics of Dimensional Logic

To establish dimensional logic as a deductive system, we need axioms that both preserve the classical structure within each dimension and regulate the transitions between dimensions.

4.1 Axioms within D_2

Within the systematic dimension, D_2 all the classical axioms of propositional and predicate logic apply (Hilbert & Ackermann, 1928). Here we give the Hilbert axioms for propositional logic in dimensionally annotated form:

$$(A1)\ \varphi^{(D_2)} \to^{(D_2)} (\psi^{(D_2)} \to^{(D_2)} \varphi^{(D_2)})$$

$$(A2)\ (\varphi^{(D_2)} \to^{(D_2)} (\psi^{(D_2)} \to^{(D_2)} \chi^{(D_2)})) \to^{(D_2)} ((\varphi^{(D_2)} \to^{(D_2)} \psi^{(D_2)}) \to^{(D_2)} (\varphi^{(D_2)} \to^{(D_2)} \chi^{(D_2)}))$$

$$(A3)\ (\neg^{(D_2)}\psi^{(D_2)} \to^{(D_2)} \neg^{(D_2)}\varphi^{(D_2)}) \to^{(D_2)} (\varphi^{(D_2)} \to^{(D_2)} \psi^{(D_2)})$$

In addition, the quantifier axioms:

$$(A4)\ \forall x\ \varphi(x)^{(D_2)} \to^{(D_2)} \varphi(t)^{(D_2)} \text{ (for substitution free Terms } t\text{)}$$

$$(A5)\ \varphi^{(D_2)} \to^{(D_2)} \forall x\ \varphi^{(D_2)} \text{ (if x is not free in } \varphi\text{)}$$

These axioms ensure that within , D_2 the full expressiveness and deductive power of classical logic is preserved.

4.2 Dimensional axioms of coherence

The relationships between the dimensions are regulated by special axioms:

Forward Compatibility:

$$(D1)\ \varphi^{(d_i)} \wedge \text{Condition}\ (\uparrow) \vdash \varphi^{(d_{i+1})} \text{ for } i < 4$$

This axiom states that a statement that applies to a lower dimension can in principle be transformed into a higher dimension, provided that the transitional conditions are met. The conditions for are context-dependent: to get from ↑ D_2 to, a reflexive structure must be specified; to get from to, a cultural-paradigmatic context must be made explicit.$D_3 D_3 D_4$

Reflexive extension:

$$(D2)\ \varphi^{(D_2)} \rightarrow \exists R: \varphi^{(D_3,R)}$$

In principle, every systematic statement can be reflexively expanded. This reflects the epistemological insight that formal systems never operate in isolation, but can always be embedded in metatheoretical considerations.

Contextual embedding:

$$(D3)\ \varphi^{(D_3,R)} \rightarrow \exists C: \varphi^{(D_4,C)} \text{ whereby } R \subseteq C$$

Every reflexive statement can be embedded in a paradigmatic context. The context encompasses the reflexive structure CR, but goes beyond it by also addressing the historical and cultural prerequisites.

Dimensional consistency:

$$(D4)\ \neg(\varphi^{(d_i)} \wedge^{(d_i)} \neg^{(d_i)} \varphi^{(d_i)}) \text{ for all } i$$

The prohibition of objection applies within every dimension. This ensures that each dimension is logically consistent in itself.

Dimensional independence:

$$(D5)\ \varphi^{(d_i)} \wedge \neg\varphi^{(d_j)} \text{ is permissible for } i \neq j$$

Between different dimensions, a statement can have different truth values without this being a contradiction. This is a fundamental difference from classical logic and reflects epistemic plurality.

4.3 Axioms of Transition

The transition operators are governed by specific axioms:

Projection axiom:

$$(T1)\ \varphi^{(d_j)} \vdash\downarrow (d_j \to d_i)(\varphi)^{(d_i)} \text{ with loss of information } \delta$$

A statement can be projected from a higher dimension to a lower one, but this is necessarily accompanied by a loss of information. δ quantifies this loss.

Axiom of reflection:

$$(T2)\ \varphi^{(D_2)} \wedge \text{Reflection}(R) \vdash \varphi^{(D_3,R)}$$

A formal statement can be nullified by specifying a reflexive structure.D_3

Contextualization axiom:

$$(T3)\ \varphi^{(D_3,R)} \wedge \text{Context}(C) \vdash \varphi^{(D_4,C)}$$

A reflexive statement can be suspended by explication of the paradigmatic context.D_4

Integration principle:

$$(T4)\ \varphi^{(d_i)} \wedge \psi^{(d_j)} \vdash \bigoplus^{\max(d_i,d_j)} (\varphi, \psi)$$

Statements from different dimensions can be integrated, whereby the target dimension is at least as high as the highest initial dimension.

5. Inference Rules

Dimensional logic preserves the classic inference rules within each dimension, but adds new rules for transitions between dimensions.

5.1 Within a dimension

Modus Ponens (dimensional):

$$\frac{\varphi^{(d)}, \varphi^{(d)} \to^{(d)} \psi^{(d)}}{\psi^{(d)}}$$

This rule applies unchanged within each dimension. It is the backbone of deductive inference.

Universal generalization:

$$\frac{\varphi^{(d)}(x)}{\forall x\, \varphi^{(d)}(x)}$$

unless it occurs freely in the assumptions.x

Universal instantiation:

$$\frac{\forall x\, \varphi^{(d)}(x)}{\varphi^{(d)}(t)}$$

for any term that is freely tx substitutable for in.φ

These rules ensure that the D_2 full deductive power of first-level predicate logic is available within (Church, 1956).

5.2 Between dimensions

Upward Transition:

$$\frac{\varphi^{(d_i)}, \text{Condition}(\uparrow)}{\varphi^{(d_{i+1})}}$$

This rule allows the transition to a higher dimension, provided that the transition conditions are met.

Reflection rule:

$$\frac{\varphi^{(D_2)}, \text{Reflection}(R)}{\varphi^{(D_3,R)}}$$

A systematic statement is raised to the third dimension by adding a reflexive structure.

Contextualization rule:

$$\frac{\varphi^{(D_3,R)}, \text{Context}(C)}{\varphi^{(D_4,C)}}$$

where specifies the paradigmatic context that includes the reflexive structure.CR

Downward Projection:

$$\frac{\varphi^{(d_j)}, \text{Projection}(\pi)}{\pi(\varphi)^{(d_i)}} \text{ for } i < j$$

This rule allows a statement to be projected to a lower dimension, specifying the projection function that specifies what information is retained.π

5.3 Category Error Avoidance

A central rule of dimensional logic is the type consistency rule:

$$\frac{\varphi^{(d_i)} \rightarrow^{(d_j)} \psi^{(d_k)}}{Derivative} \text{ only if Transition}(d_i \rightarrow d_j) \wedge \text{Transition}(d_j \rightarrow d_k) \text{ allowed}$$

This rule prevents unacceptable dimensional jumps. You can't just link a statement from D_1 (immediate perception) to a statement from D_4 (contextual embedding) without using explicit transform operators. Violations of this rule are formally demonstrable category errors (Ryle, 1949).

6. Detailed Examples

To demonstrate the practical applicability of dimensional logic, we will work through three detailed examples: the prisoner's dilemma with concrete parameters, an ethical everyday decision, and the scientific statement about electron spin.

6.1 Example 1: The prisoner's dilemma with concrete figures

The prisoner's dilemma is the paradigmatic example of the tension between individual and collective rationality (Rapoport & Chammah, 1965). We look at the classical formulation and then show how the dimensional logic leads to a different solution.

6.1.1 Classic formulation on D_2

Two players each have two strategies: cooperate (C) or defect (D). The payout matrix is:

	Player 2: C	Player 2: D
Player 1: C	(3, 3)	(0, 5)
Player 1: D	(5, 0)	(1, 1)

This corresponds to the standard parameterization with:

- $T = 5$ (Temptation, temptation to one-sided defection)
- $R = 3$ (Reward, reward for mutual cooperation)
- $P = 1$ (Punishment, punishment for mutual defection)
- $S = 0$ (Sucker's payoff)

The utility functions can be represented formally:

$$U_1(C,C)^{(D_2)} = 3, U_1(C,D)^{(D_2)} = 0, U_1(D,C)^{(D_2)} = 5, U_1(D,D)^{(D_2)} = 1$$

$$U_2(C,C)^{(D_2)} = 3, U_2(D,C)^{(D_2)} = 0, U_2(C,D)^{(D_2)} = 5, U_2(D,D)^{(D_2)} = 1$$

The Nash equilibrium condition (Nash, 1950) requires that no player can improve his utility by one-sided deviation:

$$\forall s_1' \neq s_1^*: U_1(s_1^*, s_2^*)^{(D_2)} \geq U_1(s_1', s_2^*)^{(D_2)}$$

$$\forall s_2' \neq s_2^*: U_2(s_1^*, s_2^*)^{(D_2)} \geq U_2(s_1^*, s_2')^{(D_2)}$$

Analysis: The following applies to player 1:

- If player 2 cooperates: (defection is better)$U_1(D, C) = 5 > U_1(C, C) = 3$
- If player 2 defects: (defection is better)$U_1(D, D) = 1 > U_1(C, D) = 0$

Analogous for player 2. Defection dominates strictly. The classic solution is:

$$(D, D)^{(D_2)}$$

with the payout profile . This is inefficient, as both $(1,1)(C, C)(3,3)$ would be better off with me.

6.1.2 Reflexive extension to D_3

Dimensional logic introduces a reflexive operator $\Delta^{(2)}$ (Stegemann, 2025b):

$$\Delta^{(2)}: (s_1, s_2)^{(D_2)} \mapsto (s_1, s_2)^{(D_3,R)}$$

where denotes the reflexive expectation structure. On the other hand, the players model mutual expectations. The extended utility function is:RD_3

$$U_i^{(D_3)}(s_i, s_j, R) = U_i^{(D_2)}(s_i, s_j) + \alpha \cdot \Delta_\mu(s_i, s_j, R)$$

where indicates the weighting of the reflexive component and $\alpha \in [0,1]\Delta_\mu$ quantifies the reflexivity gain.

Definition of:Δ_μ The reflexivity gain occurs when both players coordinate their perspectives. Formal:

$$\Delta_\mu(C, C, R) = +2 \text{ (both expect cooperation)}$$

$$\Delta_\mu(D, D, R) = 0 \text{ (no positive coordination)}$$

$$\Delta_\mu(C, D, R) = -1 \text{ (Disappointment of expectation)}$$

$$\Delta_\mu(D, C, R) = -1 \text{ (Disappointment of expectation)}$$

Numerical evaluation with :$\alpha = 0.4$

$$U_1^{(D_3)}(C, C, R) = 3 + 0.4 \cdot 2 = 3.8$$

$$U_1^{(D_3)}(D, D, R) = 1 + 0.4 \cdot 0 = 1.0$$

$$U_1^{(D_3)}(C, D, R) = 0 + 0.4 \cdot (-1) = -0.4$$

$$U_1^{(D_3)}(D,C,R) = 5 + 0.4 \cdot (-1) = 4.6$$

New equilibrium analysis:

- If Player 2 cooperates: (Defection still better)$U_1^{(D_3)}(D,C) = 4.6 > U_1^{(D_3)}(C,C) = 3.8$
- If player 2 defects: (defection still better)$U_1^{(D_3)}(D,D) = 1.0 > U_1^{(D_3)}(C,D) = -0.4$

With only a reflexive component, defection remains dominant, but the advantage has shrunk.

6.1.3 Contextual Integration on D_4

Now let's add the contextual operator $\Delta^{(3)}$:

$$\Delta^{(3)}: (s_1, s_2)^{(D_3,R)} \mapsto (s_1, s_2)^{(D_4,C)}$$

where C denotes a normative context (e.g. fairness norm, culture of trust). The full utility function is:

$$U_i^{(D_4)}(s_i, s_j, R, C) = U_i^{(D_2)}(s_i, s_j) + \alpha \cdot \Delta_\mu(s_i, s_j, R) + \beta \cdot \Delta_\kappa(s_i, s_j, C)$$

where is the weighting of the contextual component and quantifies the gain in context.$\beta \in [0,1]\Delta_\kappa$

Definition of:Δ_κ In a cooperative context, additional gains arise from reputation, reciprocity, and social norms (Ostrom, 1990; Fehr & Gächter, 2000):

$$\Delta_\kappa(C,C,C) = +1.5 \text{ (both meet standard)}$$

$$\Delta_\kappa(D,D,C) = -1.0 \text{ (both violate norm)}$$

$$\Delta_\kappa(C,D,C) = +0.5 \text{ (Moral gain despite loss)}$$

$$\Delta_\kappa(D,C,C) = -2.0 \text{ (Serious violation of norms)}$$

Numerical evaluation with :$\alpha = 0.4, \beta = 0.3$

$$U_1^{(D_4)}(C,C,R,C) = 3 + 0.4 \cdot 2 + 0.3 \cdot 1.5 = 4.25$$

$$U_1^{(D_4)}(D,D,R,C) = 1 + 0.4 \cdot 0 + 0.3 \cdot (-1.0) = 0.7$$

$$U_1^{(D_4)}(C,D,R,C) = 0 + 0.4 \cdot (-1) + 0.3 \cdot 0.5 = -0.25$$

$$U_1^{(D_4)}(D,C,R,C) = 5 + 0.4 \cdot (-1) + 0.3 \cdot (-2.0) = 4.0$$

Final equilibrium analysis:

- If Player 2 cooperates: (Cooperation is better!)$U_1^{(D_4)}(C,C) = 4.25 > U_1^{(D_4)}(D,C) = 4.0$
- If player 2 defects: (defection is better)$U_1^{(D_4)}(D,D) = 0.7 > U_1^{(D_4)}(C,D) = -0.25$

Now there are **two Nash equilibria**: with payoff and with payoff.$(C,C)4.25(D,D)0.7$

The cooperative balance is not only Nash-stable, but also Pareto-dominant and delivers the highest payout.

6.1.4 Formal derivation

The full dimensional derivative is:

1. $\text{Def(PD)}^{(D_2)} \rightarrow^{(D_2)} (D,D)^{(D_2)}$ (Standard Nash Equilibrium)
2. $\text{Def(PD)}^{(D_2)}$ (Acceptance)
3. $(D,D)^{(D_2)}$ (Modus Ponens, 1, 2)
4. $\text{Reflexion}(R){:}\,\Delta_\mu(C,C) = +2$ (Reflexive Structure Specification)
5. $(s_1,s_2)^{(D_3,R)}$ (Rule of Reflection, 3, 4)
6. $\text{Context}(C){:}\,\Delta_\kappa(C,C) = +1.5$ (Specification of normative context)
7. $(C,C)^{(D_4,C)}$ (Contextualization Rule, 5, 6)
8. $U^{(D_4)}(C,C) = 4.25 > U^{(D_4)}(D,D) = 0.7$ (Numerical evaluation)
9. $(C,C)^{(D_4,C)}$ is Pareto-dominant Nash equilibrium (Conclusion)

Interpretation: Dimensional logic formally shows how cooperation is rationally stabilized by integrating reflexive and contextual components. This is not an external additional assumption, but a structural consequence of dimensionally expanded thinking (Axelrod, 1984).

6.2 Example 2: Ethical Everyday Decisions

Let's consider a concrete ethical decision and formalize it dimensionally: "Should I help my colleague with his presentation, even though I jeopardize my own deadline?"

6.2.1 Formulation to : Immediate reactionD_1

Operated on Immediate Perception and Affective Evaluation:D_1

$$\text{Help}(K)^{(D_1)} \equiv \text{Empathy}(K) \wedge \text{Load(I)}$$

The direct experience shows: I feel empathy for the colleague (he is stressed) and at the same time my own burden (time pressure). At this level, there is still no propositional structure, only phenomenal presence of tension.

Truth Profile: (moderate immediate inclination to help).$V(\text{Help})^{(D_1)} = 0.6$

6.2.2 Systematic analysis of D_2

The D_2 situation is formally structured. We define predicates:

- $H(x,y)$: "x helps "y
- $N(x)$: "fulfills its obligations"x
- $U(x,a)$: "Benefit of Action for ax"

The systematic decision is based on benefit maximization:

$$U(\text{Ich},H)^{(D_2)} = -2 \text{ (Costs due to loss of time)}$$

$$U(\text{Colleague},H)^{(D_2)} = +3 \text{ (Benefits for colleagues)}$$

$$U_{\text{total}}^{(D_2)} = -2 + 3 = +1$$

Formally, the following applies:

$$U_{\text{total}}^{(D_2)} > 0 \rightarrow^{(D_2)} H(\text{I},\text{Colleague})^{(D_2)}$$

Systematic recommendation: Helping is rational because the overall benefit is positive.

Truth profile: (clear under utility maximization).$V(\text{Help})^{(D_2)} = 1.0$

6.2.3 Reflexive consideration of D_3

Let D_3 's add reflexive considerations:

- "What does my colleague think about my motives when I help?"
- "What do I think about my decision afterwards?"
- "How does this decision affect our future cooperation?"

The reflexive structure includes mutual expectations and metacognitive evaluation:R

$$\Delta_\mu(H,R) = +1.5 \text{ (Trust is strengthened, reciprocity established)}$$

The extended utility function:

$$U^{(D_3)}(\text{Help},R) = U^{(D_2)}(\text{Help}) + \Delta_\mu(H,R) = 1 + 1.5 = 2.5$$

Reflexive insight: Helping creates a positive feedback loop in the relationship, making future cooperation more likely (Tomasello, 2019).

Truth profile: $V(\text{Help})^{(D_3,R)} = 1.0$ (reflexively reinforced).

6.2.4 Contextual embedding on D_4

The decision is embedded in a normative and cultural context:D_4

- Corporate culture: Is helpfulness appreciated or interpreted as a weakness?
- Professional norms: Is there an implicit expectation of mutual support?
- Power structures: Does the decision affect my position in the team?

The context C is: "Collaborative corporate culture with norm of mutual support".

$$\Delta_\kappa(H, C) = +1.0 \text{ (positive Reputation, Compliance with standards)}$$

Full benefit function:

$$U^{(D_4)}(\text{Help}, R, C) = U^{(D_2)}(\text{Help}) + \Delta_\mu(H, R) + \Delta_\kappa(H, C) = 1 + 1.5 + 1.0 = 3.5$$

Contextual insight: In a collaborative culture, helpfulness is rewarded not only individually, but also structurally.

Truth profile: (contextually maximally justified).$V(\text{Help})^{(D_4,C)} = 1.0$

6.2.5 Dimensional Truth Profile of the Decision

The full analysis reveals:

$$V(\text{Help}) = (0.6, 1.0, 1.0, 1.0)$$

The decision to help is justified on all levels above rational. Only the immediate affective reaction shows ambivalence.D_1

Formal derivation:

1. $U^{(D_2)}(\text{Help}) = +1^{(D_2)}$ (Systematic analysis)
2. Reflexion(R): $\Delta_\mu = +1.5^{(D_3,R)}$ (Reflexive Extension)
3. $U^{(D_3)}(\text{Help}, R) = 2.5^{(D_3,R)}$ (Integration D2 + D3)
4. Context(C): $\Delta_\kappa = +1.0^{(D_4,C)}$ (Contextual Extension)
5. $U^{(D_4)}(\text{Help}, R, C) = 3.5^{(D_4,C)}$ (Full integration)
6. $U^{(D_4)}(\text{Help}, R, C) > 0 \rightarrow H(\text{I,Colleague})^{(D_4,C)}$ (Decision rule)
7. $H(\text{Ich,Colleague})^{(D_4,C)}$ (Modus Ponens, 5, 6)

6.3 Example 3: Scientific statement (electron spin)

Let's consider the statement "electrons have spin $\frac{1}{2}$" and analyze it dimensionally.

6.3.1 Wording on : Formal systematicsD_2

On the systematic level, this is a well-formulated statement of quantum mechanics:

$$\mathrm{Spin}(e,\frac{1}{2})^{(D_2)}$$

This statement follows from the postulates of quantum mechanics. Formally, it can be deduced:

1. Postulate(QM): $\hat{S}^2|s,m\rangle = \hbar^2 s(s+1)|s,m\rangle^{(D_2)}$ (Spin Operator Axiom)
2. Empirical data: Stern-Gerlach experiment shows splitting into 2 states$^{(D_2)}$
3. Number of states $= 2s+1 = 2 \rightarrow s = \frac{1}{2}^{(D_2)}$ (Mathematical inference)
4. $\mathrm{Spin}(e,\frac{1}{2})^{(D_2)}$ (Modus Ponens, 1, 2, 3)

Within the axiomatic structure of quantum mechanics, the statement is true:

$$\mathcal{M}_{\mathrm{QM}} \vDash \mathrm{Spin}(e,\frac{1}{2})^{(D_2)}$$

Truth profile: (formally and systematically true).$V(\mathrm{Spin})^{(D_2)} = 1.0$

6.3.2 Reflexive consideration of D_3

On one can see the metatheoretical conditions:D_3

$$\mathrm{Spin}(e,\frac{1}{2})^{(D_3,R)}$$

The reflexive structure includes:R

- The measurement theory: Spin is not a property independent of measurement, but is constituted by measurement
- The role of the observer: Quantum mechanics requires a distinction between system and measuring apparatus
- The construction aspects: "Spin" is a theoretical construct, not a classical observable

Reflexive insight: The statement $\mathrm{Spin}(e,\frac{1}{2})$ is true, but its truth depends on a specific metatheoretical framework (Heisenberg, 1958). "Spin" does not exist as a property like "position" in classical mechanics.

Formally, we can express this:

$$\mathrm{Spin}(e,\frac{1}{2})^{(D_3,R)} \iff [\mathrm{Spin}(e,\frac{1}{2})^{(D_2)} \wedge \text{Measurement theory}(\mathrm{QM})^{(D_3,R)}]$$

Truth profile: $V(\mathrm{Spin})^{(D_3,R)} = 0.9$ (reflexively true, but with epistemological restriction).

6.3.3 Contextual embedding on D_4

On the paradigmatic embeddedness is discussed:D_4

$$\mathrm{Spin}(e,\frac{1}{2})^{(D_4,C_{\mathrm{QM}})}$$

The context includes:C_{QM}

- The paradigm of quantum theory (vs. classical mechanics)
- Die historische Entwicklung (Stern-Gerlach 1922, Pauli 1925, Dirac 1928)
- The Interpretation (Copenhagen Interpretation, Many Worlds, Bohmian Mechanics)
- The cultural prerequisites (acceptance of non-classical characteristics)

In the context of classical mechanics, "spin" is not even defined:

$$\mathrm{Spin}(e,\frac{1}{2})^{(D_4\mathrm{classical})} = \text{undefined}$$

So the statement is context-relative. Formal:

$$\mathrm{Spin}(e,\frac{1}{2})^{(D_4,C_{\mathrm{QM}})} \wedge \neg\mathrm{Spin}(e,\frac{1}{2})^{(D_4,C_{\mathrm{classical}})}$$

This is not a contradiction, but an explication of contextual relativity (Kuhn, 1962).

Truth profile: $V(\mathrm{Spin})^{(D_4,C_{\mathrm{QM}})} = 1.0$ (fully valid in the QM context), but (invalid in the classical context).$V(\mathrm{Spin})^{(D_4,C_{\mathrm{Classical}})} = 0.0$

6.3.4 Full dimensional derivation

1. $\mathrm{Postulat(QM)}^{(D_2)} \to^{(D_2)} \mathrm{Spin}(e,\frac{1}{2})^{(D_2)}$ (Axiom)
2. $\mathrm{Postulat(QM)}^{(D_2)}$ (Acceptance)
3. $\mathrm{Spin}(e,\frac{1}{2})^{(D_2)}$ (Modus Ponens, 1, 2)
4. $\mathrm{Reflexion}(R)\text{:}\mathrm{Messtheorie(QM)}$ (Metatheoretical condition)
5. $\mathrm{Spin}(e,\frac{1}{2})^{(D_3,R)}$ (Rule of Reflection, 3, 4)
6. $\mathrm{Kontext}(C_{\mathrm{QM}})\text{:}\mathrm{Paradigma(Quantum\ theory)}$ (Paradigmatic embedding)
7. $\mathrm{Spin}(e,\frac{1}{2})^{(D_4,C_{\mathrm{QM}})}$ (Contextualization Rule, 5, 6)
8. $\mathrm{Kontext}(C_{\mathrm{Classical}})\text{:}\mathrm{Paradigma(Classical\ Mechanic)}$ (Alternative embedding)
9. $\neg\mathrm{Spin}(e,\frac{1}{2})^{(D_4,C_{\mathrm{Classical}})}$ (Context Relativity)

10. $\mathrm{Spin}(e,\frac{1}{2})^{(D_4,C_{\mathrm{QM}})} \wedge \neg\mathrm{Spin}(e,\frac{1}{2})^{(D_4,C_{\mathrm{Classical}})}$ (No inconsistency due to dimensional independence)

Dimensional Truth Profile:

$$V(\mathrm{Spin}) = (0.2,1.0,0.9,1.0 \mid C_{\mathrm{QM}})$$

The statement is D_1 barely accessible (spin is not immediately perceptible), formally D_2 true, D_3 reflexively true with restrictions, and completely true in the quantum mechanical context.D_4

7. Formal properties of the system

Dimensional logic has several remarkable meta-logical properties that distinguish it from classical logic.

7.1 Completeness within D_2

Within the systematic dimension, D_2 the classic completeness result applies (Gödel, 1930). If a statement follows semantically from a set of premises, then it can also be derived syntactically:

$$\Gamma^{(D_2)} \vDash \varphi^{(D_2)} \implies \Gamma^{(D_2)} \vdash \varphi^{(D_2)}$$

This follows directly from Gödel's completeness theorem for first-level predicate logic, since structurally is equivalent to classical logic.D_2

7.2 Dimensional Relativity of Completeness

Between different dimensions, completeness no longer applies in this form. It is possible for a statement to follow semantically on one dimension but not to be derived on another:

$$\varphi^{(d_i)} \vDash \psi^{(d_j)} \nRightarrow \varphi^{(d_i)} \vdash \psi^{(d_j)} \text{ for } i \neq j$$

This is because the transitions between dimensions require additional conditions that cannot be derived purely formally. The specification of a reflexive structure or paradigmatic context involves epistemic decisions that cannot be made algorithmically.

7.3 Consistency

The system is consistent within each dimension:

$$\Gamma^{(d_i)} \nvdash (\varphi^{(d_i)} \wedge^{(d_i)} \neg^{(d_i)} \varphi^{(d_i)})$$

for all . This follows from the consistency of classical logic for iD_2 and the definition of the other dimensions as epistemic extensions without modification of the logical core structure.

However, between dimensions, there can be apparent "contradictions" that are not real contradictions:

$$\varphi^{(D_2)} \land \neg\varphi^{(D_4,C)}$$

is consistent, as the statements operate on different epistemic levels. For example, a statement can be formally and systematically true, but not valid in a specific paradigmatic context (as shown in the electron spin example).

7.4 Decidability

The decidability of dimensional logic depends on the dimension. In , decidability is analogous to classical logic: propositional logic is decidable (Post, 1921), first-level predicate logic is semi-decidable (recursively enumerable), but not decidable (Church, 1936).D_2

For D_3 and , the situation becomes more complex, since the specification of reflexive structures and paradigmatic contexts cannot be done algorithmically. The question of whether the reflexive structure holds true requires an analysis of the reflexive structure $D_4\varphi^{(D_3,R)}R$that goes beyond purely formal procedures.

7.5 Category Accuracy

A central formal property of dimensional logic is the provability of category errors (Ryle, 1949). A category error occurs when a statement from one dimension without a valid transition is linked to a statement from another dimension:

$$\varphi^{(d_i)} \rightarrow^{(d_j)} \psi^{(d_k)} \text{ is not allowed if there is no transition}$$

The system can formally identify such errors by checking whether the required transition operators are present. This makes dimensional logic a tool of epistemic critique: it can prove where inadmissible dimensional jumps exist in an argumentation.

8. Comparison with other logic systems

Dimensional logic stands in a tradition of extensions of classical logic, but is fundamentally different from existing approaches.

8.1 Multi-valued logics

Multivalued logics such as Łukasiewicz's three-valued logic (1920) or fuzzy logic (Zadeh, 1965) extend the set of truth values beyond {true, false}. Dimensional logic does not do this. It retains the bivalentity within each dimension, but additionally introduces dimensional profiles. The difference is that multivalued logics make truth gradual, while dimensional logic makes truth perspectival.

8.2 Modal logics

Modal logics introduce operators such as □ (necessary) and ◊ (possible) to capture different modes of validity (Kripke, 1963). Dimensional logic has structural similarities, as different modes are distinguished here as well. The crucial difference is that modal operators operate within a uniform semantic framework (Kripke structures), while dimensional operators mediate between categorically different epistemic levels.

8.3 Context Logics

Contextual logics, such as those developed by McCarthy (1993), formalize the dependence of statements on contexts. Dimensional logic integrates contextuality as one of four dimensions (), but goes beyond this by formally $D_4 D_1$distinguishing between immediate perception (), systematic structure (), and reflexive metacognition ().$D_2 D_3$

8.4 Intuitionistic and Paraconsistent Logics

Intuitionistic logic modifies the conditions of validity for classical operators (Brouwer, 1923). Paraconsistent logics allow local contradictions without global collapse (Priest, 2006). Dimensional logic retains the classical structure within , but relativizes validity between dimensions, so that apparent contradictions can be understood as dimensional differences.D_2

8.5 Comparison Table

Aspect	Standard Predicate Logic	Dimensional logic
Truth	binary (T/F)	dimensionally relativized (v_1, v_2, v_3, v_4)
Reflexivity	Tarski hierarchy (formal)	internal in (epistemic)D_3
Context	implicit/ignored	explicitly in D_4
Inference	one-dimensional	multidimensional with transitions
Category Errors	syntactic type errors	Epistemic dimensional errors
Completeness	yes (in)D_2	dimensional relative

9. Conclusion

Dimensional logic represents a systematic extension of classical predicate logic, which formally integrates epistemic stratification. It answers the question posed at the beginning about the formalizability of dimensional epistemology in the affirmative: Yes, dimensional theory can be formulated in the language of predicate logic, but with essential extensions.

These extensions concern syntax (dimensionally typed terms and formulas), semantics (dimensionally layered models and truth profiles), axiomatics (coherence and transition axioms), and inference rules (transition rules between dimensions). The resulting system retains the strengths of classical logic within the systematic dimension D_2, but expands the framework to include immediate perception (D_1), reflexive metacognition (D_3), and contextual embedding (D_4).

The detailed examples have shown how dimensional analysis works in practice. In the prisoner's dilemma, the integration of reflexive and contextual operators with concrete parameters () leads to a cooperative Nash equilibrium with payoff 4.25, which dominates the defective equilibrium with payoff 0.7 (Axelrod, 1984; Ostrom, 1990). In everyday ethical decisions, the dimensional truth profile allows for a nuanced evaluation that combines immediate ambivalence with systematic, reflexive, and contextual justification. In scientific statements, dimensional analysis makes context relativity explicit: "Spin $\alpha = 0.4, \beta = 0.3(0.6,1.0,1.0,1.0)\frac{1}{2}$" is true in the quantum mechanical context, undefined in the classical context, without this being a contradiction (Heisenberg, 1958; Kuhn, 1962).

The practical significance of this extension lies in the ability to formally identify category errors (Ryle, 1949). If an argument jumps inadmissibly between dimensions, this can be proven to be a violation of the transition rules. Dimensional logic thus becomes a tool of epistemic critique that goes beyond purely formal analysis and makes the epistemic structure of argumentation transparent.

Future work will have to further develop the foundations laid here, in particular towards a complete proof theory, a detailed model theory and concrete applications in philosophy, philosophy of science and artificial intelligence (Stegemann, 2025b). Dimensional logic is not a finished system, but an open research framework that raises new questions and opens up new perspectives on the structure of thought.

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