Dimensional Logic — Basics, Structure and Applications

Corrected Edition: D3 (Reflexive) / D4 (Contextual)

Wolfgang Stegemann

1 Introduction

Classical logic was and is one of the most powerful instruments in the history of science and knowledge. As a formalized system for judging truth on the basis of well-formed statements, it has created a structure that guarantees clarity, deducibility and argumentative stringency. But it is precisely where it unfolds its greatest strength – in the exclusion of contradiction and the determination of binary validity – that its limits become apparent in modern, complex contexts. Whenever self-reference, contextual dependence (Hofstadter 1979), emergence or interactive dynamics come into play, classical propositional logic proves to be epistemically too narrow (Priest 2006; Haack 1978).

In areas such as ethics, consciousness research, systems theory, cognitive science or the philosophy of mind, one regularly encounters structures that either cannot be grasped in a classical logical way or whose validity cannot be thought of independently of the context or the subject. Terms such as "identity", "truth", "responsibility", "intelligence" or "meaning" cannot be conclusively defined by formal two-value. In mathematics itself – for example, in the treatment of uncertainty, undecidability (van Benthem 2010) or the embedding of mathematical statements in epistemological frameworks – it becomes clear that logic as a system is not sufficient to understand validity as a process.

The "dimensional logic" proposed here starts at exactly this point. It is not a departure from classical logic, but a structured extension of its conditions of validity (Luhmann 1990). It is based on the idea that thinking is not one-dimensional, but happens in levels of abstraction that build on each other. These levels, known as dimensions of thought, describe different modes of cognitive processing – from the immediate perception of difference to formal structuring and reflexive self-assurance to contextual embedding.

This article develops this logic from the ground up. It begins with a systematic problem, introduces the dimensions of thought and shows how classical logic fits into this structure.

Subsequently, the dimensional logic is formalized, underpinned with examples and finally presented in its mathematical extension. The aim is to create a coherent, epistemologically viable system that can be used for philosophical reflection as well as for practical applications (e.g. in decision research, AI, ethics or model theory).

The basic assumption here is that truth is not independent of its scope of validity, and validity is not independent of the structural mode of thought. Anyone who takes thinking itself seriously as a structurally changing process must also adapt the logic with which it operates to this claim. Dimensional logic is a suggestion to take this step consistently.

2 Logical Foundations and Epistemological Motivation

Classical propositional logic is based on the principle of bivalence: a proposition is either true or false, *tertium non datur*. This structure has the advantage that it can be represented completely formally and allows a clear, deductive system. In combination with set theory, modal logic or predicate logic, this resulted in a system that can logically grasp formal proofs, mathematical definitions and even technical circuits in a logically uniform way.

But this system is not without prerequisites. It is based on a fundamental abstraction: that truth can be determined independently of the subject and the context. This assumption is legitimate in many applications – in arithmetic, geometry, algorithmic structuring – but it is not tenable everywhere. As soon as it comes to meaning, interpretation, social reality or subjective experience, binary validity loses its epistemological ground.

In addition, classical logic itself is not free of problems. The well-known paradoxes (e.g. the liar), self-referentiality (e.g. in Gödel's incompleteness theorems), or semantic ambiguities (e.g. in natural language) show that there are statements that elude a clear true/false assignment – without being meaningless. In mathematics, too, statements such as the axiom of choice or the continuum hypothesis are formally undecidable – which means that their validity depends on the chosen system.

This insight is not new. Since the 20th century, alternative logic systems have developed: multivalued logics, intuitionistic logic, modal, temporal or paraconsistent logics. But all these systems remain within a formal framework – they modify validity, but not the mode of thought itself.

What is missing is an approach that epistemologically starts from the structure of thought. The thesis is: Thinking is not flat. It is layered. It moves through stages of distinction, structuring, reflexive self-assurance, and contextualization. These stages can be described as dimensions of thought. And each of these dimensions creates its own type of validity.

The task, then, is not to invent an alternative system of logic, but to integrate the existing systems into an overarching framework – one that does not come logically from the outside, but is cognitively derived from within. Dimensional logic wants to do just that: it wants to take the mode of thought itself seriously – and develop a system of validity from it that not only examines statements, but also knows their structure and their epistemic location.

3 Derivation of the Four Dimensions of Thought

The idea of dimensional logic is based on the insight that human thought is not linear, but structurally stratified. This layering corresponds to different processing modes, which differ from each other in an increasing degree of abstraction and an increasing epistemic depth. Rather than reducing all forms of thought to a unified logical grid, dimensional logic takes seriously the fact that there are qualitative transitions in thought – transitions that are also logically and formally significant.

The four dimensions of thought on which this logic is based are not arbitrarily chosen. They arise from an analysis of thought itself – as a process that oscillates between raw perception and reflected self-structure. Each dimension of thought establishes a specific type of validity that operates on a certain level of abstraction.

3.1 D1: Difference and Direct Distinction

This lowest level of thinking is pre-linguistic. Here, differences are felt, perceived, experienced. A child who distinguishes between warm and cold, light and dark, closeness and distance moves on this level. Validity in D1 means: something is experienced as different – not logically, but phenomenally. This dimension is the basis of all further abstraction.

3.2 D2: Classical Logic and Formal Structure

This is where thinking in the narrower sense begins. Distinctions become concepts, concepts become statements, statements become derivations. Classical two-valued logic operates on this level: it knows truth values, operators, axioms. D2 is the level of explicit, rule-driven structuring. Its validity results from inner consistency and deductive consistency.

Formal structure:

$$\begin{aligned}
\text{Expression space:} \quad & \Sigma_2 = \{\varphi\} \quad \text{(classical logical statements)} \\
\text{Structuring rules:} \quad & R_2 = \text{Connectives, quantifiers, proofs} \\
\text{Condition of validity:} \quad & G_2 = \text{derivable or axiomatically true}
\end{aligned}$$

3.3 D3: Reflexive Meta-Validity and Epistemic Self-Structure

The third dimension of thought reflects the conditions of validity itself. Here one asks not only whether a statement is true, but what it means that it is true – under what conditions, by what epistemic means, within which structure. In D3, thinking itself becomes an object. Validity is not only applied, but also questioned, reconstructed and stabilized. This includes the ability to change perspectives (theory of mind), to model mutual expectations, and metatheoretical reflection on one's own cognitive processes.

Formal structure:

$$\begin{aligned} \text{Expression space:} \quad & \Sigma_3 = \{G(\varphi_C)\} \quad \text{(statements about validity)} \\ \text{Structuring rules:} \quad & R_3 = \text{Self-referential meta-structures} \\ \text{Condition of validity:} \quad & G_3 = \text{coherently stabilized validity} \end{aligned}$$

The formal operator of D3 is the validity function G: it maps any statement φ to a statement about the validity conditions of φ. This means that D3 is not just another level of content but a level of epistemic self-examination. A D3-valid statement is one that has passed through this self-examination and emerged coherently stabilized: its validity conditions are known, examined, and found to be non-circular in a problematic sense.

3.4 D4: Contextual Validity and Situational Embedding

The fourth dimension recognizes that validity does not take place in a vacuum. Statements are only meaningful if they are embedded in a context. "This is courageous" – this statement has no fixed meaning without knowledge of the circumstances. In D4, contextual logic emerges: statements are valid within a social, cultural, discursive or model-theoretical framework. Truth becomes relational.

Formal structure:

$$\begin{aligned} \text{Expression space:} \quad & \Sigma_4 = \{\varphi_C\} \quad \text{(statements with context binding)} \\ \text{Structuring rules:} \quad & R_4 = \text{Relation to epistemic environments} \\ \text{Condition of validity:} \quad & G_4 = \text{applies within a context } C \end{aligned}$$

The context parameter C in D4 is not a variable in the classical sense – it is not something that is fixed and then held constant while φ varies. Rather, C and φ are mutually constitutive: the context shapes the meaning of the statement, and the statement, once uttered, modifies the context in which subsequent statements are interpreted. This mutual constitution is what makes D4 irreducible to D2 or D3: it cannot be captured by adding a context variable to a formal system, because the context-statement relationship is dynamic and self-modifying.

D4-valid statements are therefore not simply context-relativized versions of D2 statements. They are statements that have been genuinely embedded in a specific epistemic environment and whose validity is only assessable from within that environment. This is the reason why cross-cultural misunderstandings, paradigm shifts in science, and disagreements between differently situated agents are so difficult to resolve by appeal to formal argument alone: the formal argument (D2) may be impeccable, but the contextual conditions of validity (D4) differ.

These four dimensions of thought are related to each other:

$$\begin{array}{ccccccc} \text{D1} & \xrightarrow{T_{1\to2}} & \text{D2} & \xrightarrow{T_{2\to3}} & \text{D3} & \xrightarrow{T_{3\to4}} & \text{D4} \\ \text{phenomenal} & & \text{formal} & & \text{reflexive} & & \text{contextual} \end{array}$$

The transitions between these levels cannot be reduced. They mark qualitative changes in the process of validity. This is precisely why it is not enough to build reflexivity into a modal logic – it must be thought of as a logical level in its own right. The same is true of context: it is not just a meta-operator, but a different mode of producing validity.

Dimensional logic is therefore not based on an expansion of the existing logic systems, but on a verticalization of the structure of validity. It recognizes that thought itself is graded – and that each stage has its own logic, its own stability, and its own contribution to knowledge.

4 Building a Dimensional Logic

The considerations so far lead to the central assumption of dimensional logic: Each mode of validity is bound to a specific dimension of thought. Statements, terms or models cannot be completely captured in a single dimension, but must be analysed in relation to their horizon of validity and located according to the situation. Dimensional logic is thus not a closed calculation, but an architecture of epistemic perspectives.

The aim is to model the transitions between the dimensions of thought in such a way that both the inner coherence of the respective level is maintained and its limits become recognizable. For this purpose, so-called transition operators are introduced. These do not represent a formal substitution, but mark a change in the epistemic framework:

These operators make it clear that dimensional logic is not only a logical extension, but also an epistemological deep structure. It allows statements to be assessed not only according to their content, but also according to their cognitive anchoring.

4.1 Dimensionally Structured Validity

Every statement can be anchored in one of the four dimensions of thought. This anchoring determines not only its structure, but also its epistemic stability. A statement that has only D2 validity – i.e. is formally derivable – but lacks D3 grounding is epistemically fragile: it may be logically valid but reflexively unfounded. A statement that satisfies D3 but not D4 may be self-consistently grounded yet fail in application because it ignores the context of its use.

The full epistemic stability of a statement – its truth in the sense of dimensional logic – is reached when it satisfies the validity conditions of all four dimensions simultaneously:

$$\text{Stable}(\varphi) \iff G_1(\varphi) \wedge G_2(\varphi) \wedge G_3(G(\varphi)) \wedge G_4(\varphi_C)$$

This condition defines the **fixpoint** of dimensional logic: the point at which a statement achieves invariance across all epistemic dimensions. Below the fixpoint, partial validity is always possible – and often practically sufficient. But only at the fixpoint does a statement achieve the full epistemic weight that dimensional logic attributes to truth.

The assignment of a statement to its primary dimension is not always unambiguous. Some statements straddle dimensions: mathematical theorems are paradigmatically D2, but their applicability is D4-conditioned; ethical judgments are D4-embedded but require D3 grounding to be more than mere convention. Dimensional logic does not resolve these ambiguities mechanically but provides a structured framework for their analysis.

4.2 Implications for a New Logic Structure

These considerations give rise to the demand for a vertically layered logic. This does not consist of a new system of truth values, but of a network of perspectives of validity that must be made explicit and related to each other. Classical propositional logic is included in this structure, but not conclusive. Its rules apply, but only in the dimension in which they originated – D2.

Dimensional logic is thus a logic of transitions. It describes not only what applies, but also why it applies, where it applies, and under what conditions it no longer applies. In its structure, it approaches a systemic metalogic that not only evaluates statements, but also makes the entire structure of the spaces of knowledge visible.

The relationship between the four dimensions is not merely sequential but also hierarchical in a specific sense: higher dimensions presuppose lower ones without reducing to them. D3 presupposes D2 (reflexivity operates on formal structures) but cannot be derived from D2 alone (self-reference is not a logical operator). D4 presupposes D3 (contextual embedding requires a reflexively grounded subject position) but goes beyond it (contexts cannot be reflexively generated; they must be encountered).

This hierarchical but non-reductive relationship is what makes dimensional logic a genuine extension of classical logic rather than a mere elaboration. It preserves everything that classical logic can do while adding structural resources for what classical logic systematically fails to capture: the situatedness of knowing subjects, the self-referential dimension of cognition, and the context-dependence of validity claims.

The practical consequence is that a complete logical analysis of any complex claim requires not just D2 evaluation (is it formally valid?) but also D3 evaluation (is it reflexively grounded?) and D4 evaluation (is it contextually appropriate?). Dimensional logic provides the formal tools for all three.

4.3 Transition Operators Between Dimensions

The transitions between the thinking dimensions are modeled by special operators:

$$
\begin{aligned}
&T_{1\to 2} : \text{Transformation of a distinction into a formal statement}\\
&T_{2\to 3} : \text{Reflection: } \varphi \Rightarrow G(\varphi)\\
&T_{3\to 4} : \text{Contextualization: } G(\varphi) \Rightarrow G(\varphi_C)\\
&T_{4\to 4} : \text{Stable self-application (fixed point)}
\end{aligned}
$$

These operators do not mark a formal substitution, but a change in the epistemic framework.

4.4 Formal Axiomatics of Dimensional Logic

Each dimension has: an expression space Σ_n, structuring rules R_n, conditions of validity G_n, and transition operators $T_{n\to n+1}$.

Dimension	Σ_n	G_n	$T_{n\to n+1}$
D1	perceptions	phenomenally given	$T_{1\to 2}$: abstraction
D2	$\{\varphi\}$	derivable	$T_{2\to 3}$: reflection
D3	$\{G(\varphi_C)\}$	stabilized	$T_{3\to 4}$: contextualization
D4	$\{\varphi_C\}$	within context C	$T_{4\to 4}$: fixed point

5 Mathematical Extension — Dimensional Mathematics

The dimensions of thought developed so far not only form an epistemological structure, but also open up the possibility of a new mathematical interpretation. While classical mathematics – like classical logic – operates at the D2 level, dimensional mathematics can enrich this basic structure with reflexive and contextual extensions.

To illustrate this extension, let's first look at how mathematical structures transform through the different dimensions of thought.

5.1 D2 — The Classical Function

Example: $f(x) = x^2$

This function is deterministic, universally valid, regardless of context, subject, or application. Its truth lies in its formal definability within a closed axiom system.

$$f : \mathbb{R} \to \mathbb{R}, \quad x \mapsto x^2$$

The derivation is unique and context-free:

$$f'(x) = 2x$$

These properties – determinism, universality and freedom from context – make D2 mathematics precise and predictable, but at the same time limit its applicability to complex, situational contexts. All further dimensions must presuppose D2 as their formal backbone while extending it with additional epistemic structure.

5.2 D3 — The Reflexive Function

The transition to the third dimension produces the most conceptually demanding extension: here the function itself becomes the object of its own analysis. Mathematically, this means that the conditions of validity of the function enter into the function as parameters.

In D3, the validity structure itself becomes part of the mathematical definition:

$$f_{D3}(x) = f\big(x \,\big|\, G(f)\big)$$

The validity structure of the function itself, e.g. its model frame, epistemic condition or internal structure, is $G(f)$.

This validity structure includes the epistemic conditions under which the function operates: its limits of accuracy, areas of applicability, and inherent model assumptions.

The result is differentiated:

$$\frac{d}{dx} f\big(x \,\big|\, G(f)\big) = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial G} \cdot \frac{dG}{dx}$$

The decisive difference to classical analysis lies in the second term of this derivation.

This derivation is no longer of a purely formal nature, but depends on the change in the structure of validity – e.g. through new insights, model changes or paradigmatic transformations.

This form of self-referentiality can also be transferred to linear systems, which leads to novel matrix structures.

5.3 D4 — The Contextual Function

The transition to the fourth dimension requires mathematical modeling of context dependency. This is done by introducing explicit context parameters that modify the validity of the function.

The introduction of a context parameter C changes the validity of the function:

$$f_C(x) = x^2 + \delta(C)$$

Here $\delta(C)$ is a modulation that represents the influence of contextual variables such as environmental conditions, measurement framework, social setting, or subjective perspectives.

The term $\delta(C)$ is not arbitrary, but represents measurable influences of the respective application framework. Its mathematical treatment requires an extension of the classical differential calculus:

$$f'_C(x) = \frac{d}{dx}\left[x^2 + \delta(C)\right] = 2x, \quad \text{if } \delta(C) \text{ constant.}$$

However, this consistency is often not given. In many practical applications, the context itself varies with the independent variable, resulting in more complex functional dependencies.

Context can also be dynamically modeled, e.g. as a function of x:

$$\delta(C(x)) \;\rightarrow\; f_C(x) = x^2 + \sin(x) \quad \text{or} \quad x^2 + \alpha \cdot x$$

5.4 Dimensional Matrices and Operators

The extension to multidimensional mathematical structures follows the same logic as for scalar functions. Contextual embedding here means that the matrix elements themselves become context-dependent.

In D4: Matrices with contextual elements:

$$A_C = \begin{pmatrix} a_{11}(C) & a_{12}(C) \\ a_{21}(C) & a_{22}(C) \end{pmatrix}$$

Application, for example, to contextual systems, such as climate models or social networks, where the strength of interactions depends on the broader framework in which the system is embedded. The mathematical treatment requires a calculus of context-dependent matrices:

$$\frac{\partial A_C}{\partial C} = \begin{pmatrix} \partial_C a_{11} & \partial_C a_{12} \\ \partial_C a_{21} & \partial_C a_{22} \end{pmatrix}$$

The reflexive dimension requires an even more fundamental transformation of the matrix structure. In D3: Reflexive matrices:

$$A_{D3} = A + \Lambda(A)$$

where $\Lambda(A)$ e.g. measures the symmetry deviation or validity stability.

Here, the matrix modifies its own structure based on continuous self-evaluation – a principle that is particularly relevant in adaptive systems. The eigenvalue structure of A_{D3} depends not only on the original matrix A but on the self-evaluation operator Λ:

$$A_{D3}\mathbf{v} = \lambda\mathbf{v} \quad \Leftrightarrow \quad (A + \Lambda(A))\mathbf{v} = \lambda\mathbf{v}$$

This means that the characteristic polynomial of A_{D3} is itself a function of the matrix A, introducing a level of self-reference into the linear algebra that has no counterpart in classical matrix theory. In practice, $\Lambda(A)$ can be implemented as a correction term derived from the deviation of A from some stability criterion – for example, the deviation from positive definiteness or from a target eigenvalue distribution.

5.5 Application Examples Through All Dimensions

Example 1: Physical principle — Law of inertia

D1 : Perception of motion or standstill

D2 : Formal law: $\sum F = 0 \Rightarrow v = \text{constant}$

D3 : Inertia as constitutive theory structure: $G(\varphi_C)$

D4 : $(\sum F = 0 \Rightarrow v = \text{constant})_C$ only valid in inertial frames

Example 2: Entropy

D1 : Impression of order/disorder

D2 : $S = k_B \ln W$ (Boltzmann) or $\Delta S = \int \frac{\delta Q}{T}$ (Clausius)

D3 : $S_{D3} = G(\varphi_C)$ — Entropy as measure of epistemic indistinguishability

D4 : Shannon-Entropy: $H = -\sum p_i \log p_i$ (observer-dependent)

Example 3: Ethical decision-making

A doctor decides whether to give an analgesic but life-shortening drug:

D1 : Immediate Perception of Suffering vs. Life

D2 : Rational Cost-Benefit Analysis according to Formal Criteria

D3 : Reflection on the Foundations of Medical Responsibility

D4 : Embedding in Medical, Legal, Cultural Context

Mathematically modelable as a multidimensional decision function: $d(x, G(d), C)$

This notation illustrates how a single decision simultaneously integrates reflexive self-assurance $G(d)$ and contextual embedding C.

6 Applications and Epistemological Implications

The current representation of dimensional logic can be applied to various areas of human cognition, decision-making, and modeling. Its strength lies in its ability not only to formally examine statements, but also to grasp their reflexive depth and contextual embedding. This opens up new possibilities for questions where classical logic or decision models reach their limits.

6.1 Decision-Making in Complex Situations

Classical decision theories are mostly based on maximizing a benefit within given alternatives. But many decisions – especially in the ethical, medical or political field – elude this scheme. They are not only factually bound, but also normative, emotional, contextual and reflexive.

Dimensional logic allows such decisions to be understood as processes that pass through several levels of validity. A decision that is only D2-optimal (formally maximizes a utility function) but fails D3 (has not been reflexively examined for its normative foundations) or D4 (ignores the specific deployment context) is at best incomplete and at worst harmful.

The dimensional structure of a well-formed decision can be represented as:

$$d_{\text{full}}(x) = d_{D2}(x) \xrightarrow{T_{2\to 3}} d_{D3}(x) \xrightarrow{T_{3\to 4}} d_{D4}(x, C)$$

Each transition marks an epistemic deepening: from formal optimization to reflexive grounding to contextual appropriateness.

A rational model on D2 that ignores D3 and D4 remains inadequate – and may even fail in practice. Dimensionally structured decision-making functions offer a more precise set of instruments here. They do not replace formal decision theory but situate it within its proper epistemic bounds.

6.2 Philosophy of Science and Model Critique

In classical philosophy of science, a distinction is often made between theories, models and observations. But it is rarely asked how the validity of a theory comes about – not only in the sense of empirical confirmation, but in the sense of its epistemic architecture.

Dimensional logic can serve as a meta-theory here. It makes visible on which level of thought a theory operates, which transitions it presupposes and where its limits lie. Consider a physical theory such as Newtonian mechanics: its D2 validity consists in its formal-mathematical derivability from axioms; its D3 validity consists in the reflexive understanding of what it means to apply a deterministic framework to physical phenomena; its D4 validity consists in its application within specific inertial reference frames and measurement contexts. A theory that is D2-valid but D3-unexamined is prone to category errors when extended to new domains. A theory that is D3-grounded but D4-ignorant will fail in deployment.

This view allows not only criticism, but also constructive model transformations – by systematically designing transitions between dimensions. A scientific revolution in Kuhn's sense can be understood as a D3-level transition: the existing formal apparatus (D2) is not abandoned but its reflexive foundations are reconstructed, enabling a new contextual embedding (D4).

The dimensional framework thus provides a structural language for the philosophy of science that goes beyond the traditional theory/observation distinction to include the reflexive and contextual conditions under which scientific validity is produced and maintained.

6.3 Artificial Intelligence and Machine Cognition

A central problem of current AI systems lies in their context-blind formalization. They operate almost exclusively on D2 – rules, statistics, vector spaces. Their ability to reflect (D3) is rudimentary, and to contextualize (D4) is often no more than statistical pattern matching over training data rather than genuine contextual embedding.

Dimensional logic provides a framework to analyze AI systems not only formally, but epistemologically – and to point out possible development directions. An "intelligent" machine in the full sense would not only have to simulate transitions between D2, D3 and D4, but also structurally master them. This means that it would have to be able to reflect on its role in the epistemic network (D3) and coherently justify context-sensitive decisions (D4).

A notable application of dimensional logic to AI architecture is the Artificial Local Intelligence (ALI) framework (Stegemann 2026), which maps the four dimensions onto distinct architectural components:

Dimension	Character	ALI Component
D1 (Reactive)	Immediate response, threshold logic	Causal Core (CC)
D2 (Systematic)	Rule-based, formal procedure	Ego
D3 (Reflexive)	Normative metacognition, veto	Super-Ego
D4 (Contextual)	External embedding, deployment signals	Context Modulator

The fixpoint criterion of dimensional logic is directly implemented as a decision stability metric: a 4/4 fixpoint score indicates that a given decision satisfies all four dimensional validity conditions simultaneously. This makes dimensional logic not only a theoretical framework but a practically applicable instrument for AI system design and verification.

6.4 Ethics and Moral Judgement

Moral judgments can also be reconstructed in a dimensioned way. A purely D2-based ethics (deontological rule application or utilitarian calculation) abstracts from the reflexive dimension of moral agency and the contextual dimension of moral situatedness. A purely D1-based ethics (immediate emotional response) lacks the structural stability of higher dimensions.

Dimension	Ethical Mode
D1	Immediate moral intuition, empathic response
D2	Deontological rule, utilitarian calculation
D3	Reflexive grounding of moral foundations, discourse ethics
D4	Contextual appropriateness, situational judgment

Dimensional logic here prevents ethics from being reduced to the application of rules or dissolved into mere subjectivity. It offers a structured middle position: moral judgments as stabilized transitions of validity. A morally mature judgment is one that has passed through all four dimensions: it is intuitively compelling (D1), formally defensible (D2), reflexively grounded in examined principles (D3), and contextually appropriate to the specific situation (D4). The dimensional framework does not generate moral content – it provides the structural conditions for moral epistemology.

6.5 Conclusion

The applications show that dimensional logic is not only a theoretical concept, but a practical instrument of knowledge. It allows complex problems to be analysed from their inner structure without falling into reductionism or arbitrariness. Its strength lies in the explication of the conditions of validity – not only of statements, but of decisions, theories, norms and models.

This makes it compatible with current debates in philosophy, philosophy of science, ethics, AI and systems theory – wherever validity is not a matter of course, but must be justified.

7 Conclusion and Outlook

The development of dimensional logic represents an attempt to take thinking seriously in its graded complexity and to derive from it a logical-structural system that integrates both formal consistency and epistemic self-reflection. It does not replace classical logic, but embeds it in a larger system – one that makes transitions, shifts in validity and processes of reflection formally and theoretically tangible.

The strength of this logic lies not in the replacement of existing procedures, but in the ability to limit and expand their scope. It offers a framework in which classical propositional logic, reflexive meta-logic, systemic model theory and epistemic context-embedding do not contradict each other, but can be understood as different aspects of a common space of thought.

With the introduction of dimensional operators, reflexive functions and context-sensitive structures, the possibility of reconnecting mathematical and epistemological concepts is also opened. Decisions, models, theories, and moral judgments no longer appear as isolated entities, but as the results of structurally embedded transitions between levels of validity.

The perspective that emerges from this is a constructive one: it does not call for the end of classical logic, but for its connection to a new phase of epistemic self-responsibility. A phase in which we say not only what applies – but also why it applies, in what framework, with what consequences, and under what conditions this validity is transferable or limited.

Future work on dimensional logic can focus in particular on the following aspects:

Dimensional logic is not a finished system, but an open space for thought. It marks a transition: from the calculation of validity to the architecture of validity – and from logic as the mechanics of rules to logic as an integrative self-assurance of thought.

Future work on dimensional logic can focus in particular on the following aspects: the formalization of mixed-dimension statements; the development of computational tools for dimensional analysis of natural language; the extension to five or more dimensions in domains that require additional epistemic stratification; and the empirical testing of dimensional logic as a framework for AI system evaluation and design.

What the framework already achieves is a systematic integration of intuition, formal structure, reflexivity, and contextual embedding into a single coherent logic of validity. This is not a minor extension of classical logic but a structural reconception of what it means for a statement, a decision, or a model to be true.

Epilogue: Dimensionality as an Expression of Categorical and Cognitive Logic

Dimensional logic is not only a formal structure for describing epistemic levels of validity. At the same time, it reflects two fundamental principles that are deeply rooted in the structure of our thinking and the living world:

1. **Categorical logic** – in the sense of transcendental structural principles (e.g. distinction, context, identity) that form the space of possibility of experience and cognition.
2. **Cognitive assimilation and accommodation** – in the sense of Piaget's psychological conceptualization, which aims at compression, abstraction and functional reorganization of perception.

These two levels – categorical and psychodynamic – overlap in dimensional logic. Their common basis is the principle of condensation, understood as: epistemic compression from perception to meaning and meaning to structure, whereby this structure in turn enables new forms of perception.

Principle of compression as a principle of life

Thinking condenses reality not only in order to order it formally, but also to make it energetically, functionally and symbolically connectable. This epistemic compression is not only a property of the mind, but reflects a basic principle of life itself:

- In biological systems, energetic coupling (e.g. in enzyme reactions, neuronal connections, symbolic cognition) leads to new energy levels – i.e. to emergent states with a higher density of information and reality.
- These new levels cannot be explained by linear addition, but by structural integration and qualitative transformation – a phenomenon that dimensional logic reconstructs as a vertical stratification of validity.

Thus, an essential aspect of dimensional compression lies in its energetic and cognitive efficiency. Abstraction – understood as the transformation of sensory immediacy into categorical, reflexive or contextual structures – is not a neutral act of cognition, but a strategic gain in dealing with complexity:

With the same energy consumption, abstraction enables a higher differentiation capacity, more precise perception structuring and more efficient storage of information.

This increase in efficiency is evident on several levels:

- Neurobiologically in the reduction of redundant stimulus patterns through context-dependent activation,
- Cognitively in the ability to use categorical schemas to grasp new situations more quickly (e.g. top-down recognition),
- Epistemologically in the possibility of treating statements not only as logically true/false, but of locating them in their dimensions of validity.

This makes it clear that dimensional logic is not only a model of thought, but also a functional principle of cognitive optimization that is deeply embedded in the structure of living knowledge. Abstraction here is not alienation from reality, but extended coupling with energetic stability – an evolutionary efficiency process that enables differentiation and overview at the same time.

The efficiency of knowledge E can be described in accordance with dimensional logic as a relationship between depth of validity, differentiation and use of energy:

$$E = \frac{D \cdot \Delta}{\varepsilon}$$

The following applies:

- D = Validity dimension (e.g. D1 to D4, numerically encodeable)
- Δ = degree of differentiation (information subtlety or semantic resolution)

- ε = Energy input (e.g. neuronal/metabolic effort)

This formula describes that higher cognitive efficiency is achieved by combining higher abstraction (dimension of validity) and semantic depth (differentiation) with the same or reduced use of energy.

→ If the dimension of thinking increases (e.g. from D2 to D4), the cognitive efficiency per unit of energy also increases.

Basics:

Stegemann, W. (2025). The Geometry of Thought.
`https://www.academia.edu/129523366/The_Geometry_of_Thought`

Stegemann, W. (2025). Dimensional Logic and Intelligence.
`https://www.academia.edu/129523462/Dimensional_Logic_and_Intelligence_A_New_Paradigm_for_AI_Architectures`

References:

1. Priest, G. *In Contradiction: A Study of the Transconsistent.* Oxford University Press, 2006.
2. Haack, S. *Philosophy of Logics.* Cambridge University Press, 1978.
3. van Benthem, J. *Modal Logic for Open Minds.* CSLI Publications, 2010.
4. Hofstadter, D.R. *Gödel, Escher, Bach: An Eternal Golden Braid.* Basic Books, 1979.
5. Bickhard, M.H. The interactivist model. *Synthese*, 2009, 166(3), 547–591.
6. Kauffman, S. *Investigations.* Oxford University Press, 2000.
7. Maturana, H.; Varela, F. *Autopoiesis and Cognition: The Realization of the Living.* D. Reidel, 1980.
8. Luhmann, N. *The Science of Society.* Suhrkamp, 1990.
9. Varela, F.; Thompson, E.; Rosch, E. *The Embodied Mind: Cognitive Science and Human Experience.* MIT Press, 1991.
10. Floridi, L. Open Problems in the Philosophy of Information. *Metaphilosophy*, 2004, 35(4), 554–582.
11. Gisin, N. Indeterminism in Physics, Classical Chaos and Böhmian Mechanics. *Entropy*, 2020, 22(7), 698.