Dimensional Transition Logic

The Formal Determination of Epistemic Transitions through Abstraction and Reflexive Relativization

> This article continues an earlier draft: Stegemann (2026), *Dimensional Logic as a Formal Inference System* (Zenodo, https://doi.org/10.5281/zenodo.19225035). That draft introduced four epistemic dimensions and transition operators between them, but left those operators substantively undetermined. The present article formally determines the central transition and clarifies the nature of the highest one.

Abstract

An earlier draft of dimensional logic introduced four epistemic dimensions into the formal syntax and postulated transition operators between them, but left these operators substantively undetermined. They had names and a notation, yet no specification of what they compute. The present article closes this gap for the central transition. It determines the step from the systematic to the reflexive level as quotient formation under a chosen equivalence relation and derives, for weaker relations, a degree of coherence that measures the formal rigor of the abstraction and is calculated rather than stipulated. This degree of coherence must be distinguished from the object related adequacy of an aspect. The transition to the highest level proves to be of a different kind. It is not a further abstraction, but the relinquishment of a privileged aspect. What follows is not an additional degree of truth, but a perspectival validity profile over the space of admissible aspects. Dimensional logic thereby acquires, for the first time, determinate transitions whose formal properties can be checked, together with a criterion that distinguishes the reflexive level from the relativizing one.

1. The Problem of Undetermined Transitions

Classical predicate logic operates within a fixed structure. Under an interpretation, statements receive a truth value, and rules of inference transfer this value from premises to conclusions. This structure is precise and fully formalizable. Its limit becomes apparent as soon as thought changes levels, as soon as it takes the formal structure itself as its object, compares it with others, or conceives it as one among possible structures. Classical logic provides no rules for such changes of level because it presupposes dimensional homogeneity.

An earlier draft of dimensional logic named these changes of level and assigned operators to them. It distinguished four epistemic dimensions and introduced upward, downward, and integration operators for the transitions between them. The draft thereby achieved a clear syntax, but remained undetermined at the decisive point. An operator that raises a statement from the systematic to the reflexive level was described as adding reflexive structure. Which operation is performed in doing so, and how the corresponding degree of validity is obtained, remained open. The degrees of validity of the individual levels appeared as stipulated numbers without a rule of calculation.

The present article remedies this deficiency for the central transition and at the same time clarifies the nature of the highest one. It shows that the step from the systematic to the reflexive level is a determinate formal operation, the formation of classes under a chosen aspect, and that the formal coherence of this formation can be calculated. It further shows that the step to the highest level is of a fundamentally different kind. D4 assigns no additional degree of truth, but relativizes the privileged aspect itself and thereby generates a perspectival validity profile.

2. The Epistemic Levels

The system distinguishes three constructive levels and a reflexive index above them. The levels are designated D1, D2, and D3, and the index D4, continuing the notation of the earlier draft. It will be shown that D4 is not a level of the same type.

D1 captures immediate perception of difference. At this level there are phenomenally given differences, but as yet no propositional structure. The transition from D1 to D2, the step from felt distinction to concept, is a problem in its own right and has been treated extensively; it is presupposed here as known.

D2 is the level of classical logic. Within a fixed structure, inferences are drawn and proofs are given; validity is two valued. The entire classical apparatus, syntax, Tarskian semantics, and rules of inference, remains untouched at this level.

D3 is the level of abstraction. At this level, cases are grouped into classes under an aspect. This step is the subject of Section 3.

D4 is the reflexive index over the lower levels. It marks every standpoint from which abstraction and comparison are undertaken as itself epistemic. D4 is the subject of Section 4.

3. The Transition from D2 to D3

3.1 The Aspect as an Equivalence Relation

Abstraction treats what is different as equal under a particular aspect. Formally, the aspect is represented by a binary relation on the set of cases. Let F be the set of cases available at D2. The cases may be individual objects, individual statements, or, as Section 5.2 shows, entire frameworks. An aspect is a relation $\sim$ on F that specifies which cases count as equal under that aspect.

The ideal aspect is an equivalence relation. It satisfies:

- **Reflexivity:** $x \sim x$ for all $x \in F$
- **Symmetry:** $x \sim y \Rightarrow y \sim x$
- **Transitivity:** $x \sim y \wedge y \sim z \Rightarrow x \sim z$

These three properties are the reason why abstraction at D3 is precise. An equivalence relation partitions F into disjoint classes that completely cover F. Every case falls into exactly one class, without remainder and without overlap.

3.2 The Quotient Operation

The transition from D2 to D3 is the formation of the quotient of F by $\sim$. From the set of cases and the chosen aspect there arises the set of classes, written $F/{\sim}$. Its elements are the equivalence classes

$$[x] = \{y \in F : y \sim x\}$$

An element of the quotient is an entire bundle of cases held together by the aspect.

This step is a genuine elevation and not merely a renaming, because the domain of objects changes. At D2, quantification ranges over the individual cases; at D3, it ranges over the classes. Statements can be made about a class that are not meaningfully applicable to any of its individual cases. A class has a cardinality, a relation to other classes, and a position within the quotient. At D3, predicates emerge that could not be formulated at D2.

The transition carries a parameter, the aspect $\sim$. The same set F yields different quotients under different aspects. The choice of $\sim$ contains the judgment that no mechanical procedure can take over. Once $\sim$ is fixed, quotient formation proceeds mechanically.

3.3 The Structured Class Field

Quotient formation generates three determinations in a single act.

Within each class, it renders indistinguishable what was previously different. This is **grouping**.

Between the classes, it generates distinction. The same relation that establishes equality within a class establishes difference between classes. This is **comparison**, the reverse side of grouping.

A structure of difference can be read from the classes. For the basic case, a binary measure is sufficient. Formally, on $F/\sim$ the difference measure is defined as

$$\delta([x],[y]) = \begin{cases} 0 & \text{if } [x] = [y] \\ 1 & \text{if } [x] \neq [y] \end{cases}$$

This measure captures whether there is a difference and leaves open the question of what the difference consists in. It can later be extended to a graded measure.

The result of the transition is the pair $(F/\sim, \delta)$, the structured class field. The reflexivity of level D3 consists in three things: grouping, comparing, and naming the relations, all of which are aspects of the single quotient operation.

3.4 The Degree of Coherence of Abstraction

Not every chosen aspect separates equally cleanly. The degree of coherence at D3 measures the formal discriminatory precision of an aspect and is calculated from the structure of the relation. It is therefore a measure of the relational coherence of the abstraction, not a complete measure of its epistemic validity.

The key lies in transitivity. Real aspects are often weaker. They are reflexive and symmetric but fail to be transitive. Such a relation is called a **tolerance relation**. It covers F with overlapping regions instead of partitioning it into disjoint classes. The concept goes back to Poincaré, who observed that equality in perception and measurement is not transitive because finite resolution makes neighboring magnitudes indistinguishable while more distant ones are not. Zeeman (1962) formalized the concept and gave it its name.

The degree of coherence quantifies the failure of transitivity. Let $\sim$ be a reflexive, symmetric relation on F. Consider all triples (x, y, z) with $x \sim y$ and $y \sim z$. The degree of coherence is the proportion of those for which $x \sim z$ also holds:

$$C_3(\sim) = \frac{|\{(x, y, z) \mid x \sim y \wedge y \sim z \wedge x \sim z\}|}{|\{(x, y, z) \mid x \sim y \wedge y \sim z\}|}$$

This value lies in $[0,1]$. It equals 1 exactly when $\sim$ is transitive. Only in this case is there an equivalence relation and the quotient formation described in Section 3.2 is possible in the strict sense. If $C_3(\sim) < 1$, there is, instead of a partition, an overlapping cover by tolerance regions. The value decreases to the extent that transitivity is violated.

The decisive point is that C_3 is calculated. Two persons who use the same set F and the same aspect $\sim$ obtain the same value. The formal discriminatory precision of the aspect is therefore checkable and not freely stipulated. Yet $C_3(\sim) = 1$ does not imply that the chosen aspect is epistemically adequate to the object. A perfectly transitive relation can classify according to a feature irrelevant to the epistemic question at hand. The degree of coherence therefore measures the internal rigor of the abstraction. Its object related adequacy forms a second, empirical component that is not captured by C_3 alone.

4. The Transition from D3 to D4

4.1 Why D4 Is Not an Abstraction

It is tempting to interpret the transition from D3 to D4 as a further quotient formation. This interpretation is mistaken, and its failure determines the nature of D4.

If D4 were a further abstraction, it would have to possess a privileged aspect relative to which it sorted precisely. Such a higher order aspect would itself have to be an aspect, and D4 would therefore be merely another D3 at a higher level. The characteristic property of D4, namely that the privileged perspective itself is relativized, could not be obtained in this way.

D4 differs from D3 precisely in lacking a privileged aspect. At D3, an aspect is fixed, and sorting and comparison take place relative to it. At D4, the aspects themselves become the object, and there is no longer a higher fixed aspect for comparing them. **D4 is the relinquishment of the privileged relation.**

4.2 The Reflexive Index

D4 adds no content to the abstraction; it withdraws a claim from it. Formally, D4 is an operator ρ that acts on statements of the lower levels and changes their status without altering their content. Just as in modal logic an operator precedes a sentence and shifts its mode of validity (Kripke, 1963), ρ precedes a statement and marks it as perspectival. The difference from a modal operator is that ρ does not quantify over possible worlds, but places the statement relative to the space of aspects from which it can be made.

Self application is decisive. The operator ρ is applied to every statement of the lower levels, including the statements of the theory itself. Dimensional logic is itself an aspect. Relative to its own D4 level, it is one among possible aspects. This stance, which consistently applies the reflexive index to everything, including the theory itself, is **reflexive relativism**.

4.3 The Validity Profile as a Derived Result

The relinquishment of the privileged aspect does not generate a further degree of truth at D4. It changes the form in which validity is ascribed.

At D3, an aspect $\sim$ is fixed. Statements and abstractions are assessed relative to this framework, and the formal coherence of the abstraction is determined by $C_3(\sim)$. At D4, no aspect is fixed. In place of a single relation stands the space $\mathcal{H}$ of admissible aspects. The coherence values therefore form not a single scalar, but a profile

$$\mathcal{P}_4 = \{(\sim, C_3(\sim)) : \sim \in \mathcal{H}\}.$$

This profile is not an additional truth scale. Rather, the reflexive operator ρ marks the fact that every ascription of validity is bound to an aspect. Since no aspect is privileged, there is no internal reason to privilege any one of the associated values or frameworks of judgment. Reducing the profile to a single value would presuppose an aggregation rule. Such a rule would itself again be a privileged aspect and thus a return to D3.

D4 is therefore not gradational in the sense of a many valued logic. It is perspectivally distributed. This distribution is the formal expression of the fact that there is no final standpoint.

4.4 The Boundary between D3 and D4

A precise criterion follows from the foregoing. Both levels compare. The difference lies in whether the comparer includes itself within the relativity.

At D3, classes or entire frameworks are compared from a standpoint that is taken as fixed ground. The determination is precise because it is made relative to this standpoint. At D4, this standpoint too is recognized as one aspect among possible aspects. D3 establishes that the frameworks judge differently. D4 recognizes that this determination itself is made from a perspective and has no neutral location.

Criterion: As long as comparison proceeds from a standpoint taken to be fixed, the operation is D3 and is precise relative to that standpoint. As soon as the standpoint of comparison itself is relativized, the operation is D4 and is perspectivally distributed.

5. Examples

5.1 Pure Abstraction with Full Degree of Coherence

At D2, four valid inferences are given. Three have the form of inference from the general to the particular. The fourth has the form of disjunctive elimination.

Let the logical form of inference be chosen as the aspect $\sim$. Two inferences count as equal if they have the same form. This relation is reflexive, symmetric, and transitive. The quotient consists of two classes. Each of the four inferences falls unambiguously into one of them. All triples that satisfy the antecedent also satisfy the consequent, so that

$$C_3(\sim) = 1$$

holds. The abstraction is perfectly rigorous. At D3, the predicate of the universally valid form of inference emerges, which can be asserted of the class as a whole but could not be formulated of any individual inference.

5.2 Frameworks as Cases

Consider two societies S_A and S_B as cases and a statement p. In S_A, p is taken to be true; in S_B, it is not. A truth in one society is not a truth in the other.

At D3, the two societies are placed under the aspect of their judgment concerning p. The binary difference measure yields $\delta = 1$ because the frameworks judge differently. This determination is precise. D3 states it without asking which framework is right.

The transition to D4 occurs as soon as the standpoint of the one making the determination is itself relativized. D4 adds that the location from which the determination is made is likewise not one that stands above the frameworks. The precise determination of D3 remains intact but is marked as perspectival.

5.3 Quantum Collapse

Quantum mechanics describes a system prior to measurement by means of a superposition and states that it collapses to a single value at the moment of measurement. At D2, this statement is well defined within the formalism and holds strictly. Nothing that follows alters D2.

The interpretation of collapse is disputed within physics. One line takes it to be a real event in the system, the other as an update of knowledge. This distinction between an ontic and an epistemic conception of the quantum state was formulated precisely by Harrigan and Spekkens (2010). The dispute itself is not decided here.

The contribution of dimensional logic cuts across this dispute. Both interpretations are aspects. The distinction between ontic and epistemic on which both rely is itself a line drawn under an aspect. At D4, the statement about collapse is marked as perspectival. What collapse might be outside every aspect is not an answerable question. The reflexive index merely withdraws from the physical statement the claim to be more than the most adequate ordering of a finding achieved so far.

6. Epistemological Foundation

The calculated degree of coherence and reflexive relativization combine into a unified interpretation.

There is an object that resists cognition and is not its product. Cognition has access to this object only in the form given to it through the choice of an aspect. What counts as ontology is the form the object assumes under an aspect. A form outside every aspect does not occur for cognition. Such an aspect free standpoint would be a view from nowhere, whose unattainability Nagel (1986) has analyzed. At the same time, the choice of aspect is not arbitrary because the object resists it. Two questions must be distinguished here. $C_3(\sim)$ answers the formal question of how coherently an aspect classifies. Whether that aspect holds up against the object is a question of its object related adequacy. A formally perfect classification may be epistemically unproductive, while an empirically adequate aspect under real measurement conditions may exhibit tolerance regions and thus a degree of coherence below 1. Formal coherence and object related adequacy are therefore distinct conditions of epistemic viability. The present formalism determines the first and leaves the complete formalization of the second open.

This position must be distinguished from two neighboring positions. It differs from **Kant** in abandoning the thing in itself as a separate layer. There is no aspect free in itself. The object is real as resistance, but it possesses no form accessible to cognition beyond every aspect. It differs from **Barad's onto epistemology** (2007), which likewise rejects the separation of ontology and epistemology, by moving in the opposite direction. Barad rejects epistemological relativism as the mirror image of objectivism. The position developed here retains reflexive relativism. The ontic remains the resistant object, ontology is its mapped form, and the reflexive index states that even the claim of this inseparability is itself an aspect.

7. Formal Properties

Within D2, classical logic remains unchanged. At this level, the system is a conservative extension.

At D3, validity relative to a fixed aspect is precise. The degree of coherence $C_3(\sim)$ can be calculated for every reflexive, symmetric relation. It takes the value 1 if and only if the relation is transitive and thus an equivalence relation. C_3 measures the formal quality of abstraction and must be distinguished from its object related adequacy.

At D4, no additional degree of truth is introduced. The operator ρ marks the aspect dependence of validity. Formally, a profile over the space of aspects $\mathcal{H}$ takes the place of a privileged framework. D4 is therefore perspectivally distributed, not many valued.

Apparent contradictions can arise between the levels that are not contradictions. A statement may hold under one framework and fail to hold under another without contradiction, because the validities are bound to different aspects. The dimensional independence of validity is consistent and forms the formal core of the compatibility of different frameworks.

8. Conclusion

The earlier draft named four dimensions and postulated transitions without determining them. The present article determines the central transition and clarifies the nature of the highest one.

For an equivalence relation, the step from the systematic to the reflexive level is the formation of the quotient $F/\sim$ under a chosen aspect $\sim$. It generates a structured class field $(F/\sim, \delta)$ that groups, distinguishes, and names the relations between classes. For reflexive and symmetric relations, the degree of coherence

$$C_3(\sim) = \frac{|\{(x,y,z) \mid x \sim y \wedge y \sim z \wedge x \sim z\}|}{|\{(x,y,z) \mid x \sim y \wedge y \sim z\}|}$$

measures the formal rigor of abstraction as proximity to transitivity and is calculated rather than stipulated. It measures relational coherence, not yet the object related adequacy of the chosen aspect.

The step to the highest level is of a different kind. It is the relinquishment of a privileged aspect. No further degree of truth follows from it. Instead of a privileged coherence value, there arises the profile

$$\mathcal{P}_4 = \{(\sim, C_3(\sim)) : \sim \in \mathcal{H}\}.$$

D4 is the reflexive index ρ, which marks every standpoint as epistemic and applies to the theory itself. Validity there is perspectivally distributed because no standpoint within the system can be privileged as final.

What remains open is the graded version of the difference measure δ, the complete determination of the space of aspects $\mathcal{H}$, and a formal determination of the object related adequacy of an aspect. At these points, the system remains an open framework. At its central point, it is determinate.

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