Association, Relation, and Category Errors

An Extension of Four-Dimensional Logic

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Abstract

Four-dimensional logic distinguishes four epistemic dimensions of thought: D1 as immediate and reactive linking, D2 as systematic and rule-guided relation formation, D3 as reflexive examination of the rules and categories employed, and D4 as the contextual and perspectival determination of their validity. The present paper extends this approach by developing a theory of relation formation. It starts from the assumption that thinking consists, at an elementary level, in registering differences and establishing connections. The character of these connections changes, however, with the epistemic dimension. D1 generates associations, D2 specifies relations, D3 examines their categorial and metatheoretical presuppositions, and D4 determines the context while also marking the chosen standpoint as perspectival. This yields a systematic account of a widespread error in scientific and philosophical reasoning: an initially justified association is upgraded at D2 into a strong relation without D3 examining the relevant levels of description or D4 examining the context of validity. To capture this error formally, a level operator is introduced that assigns each expression to a level of description. Relations additionally receive a signature defined as a set of admissible pairs of levels. An unmarked transfer of a relation to terms belonging to other levels of description thereby becomes visible as an unjustified relation transfer. Using functionalism as a further example, the paper shows that partial functional equivalence does not entail phenomenal equivalence unless an independent bridge premise is justified. A strong functionalism can formulate such a bridge as a claim of sufficiency or identity, but thereby shifts the burden of justification to D3. The extension remains compatible with the existing framework of dimensional logic because it introduces neither new truth values nor new ontological layers. Rather, it sharpens the epistemic control of relations and makes visible how plausible connections can develop into category errors.

1. Starting Point: Thinking as Relation Formation

Four-dimensional logic was developed as an extension of classical bivalent logic (Stegemann 2025a, 2025b, 2026). Its aim is not to replace true and false with additional truth values. The classical distinction remains the elementary form of every determinate proposition. What is extended is not the truth value, but the epistemic space within which propositions are formed, tested, reflected upon, and contextualized. In the revised version, four dimensions are distinguished: D1 as the immediate and reactive dimension, D2 as the systematic dimension, D3 as the reflexive dimension, and D4 as the contextual and perspectival dimension. D4 is not another level of abstraction. It also relativizes the standpoint from which validity is determined. The higher dimensions do not abolish the lower ones. They add further possibilities for forming and controlling relations.

This structure can be made more precise if thinking is understood, in a very general sense, as the establishment of relations. An organism must be able to register differences. Beyond this, it must be able to connect states, events, and consequences. Without this capacity there would be neither learning nor expectation nor goal-directed behavior. Even a simple organism depends on coupling certain changes in its environment with certain internal states and responses.

The most elementary form of such a connection can initially be written, without further specification, as

$$x \sim y$$

The symbol $\sim$ asserts neither causality nor logical necessity. It states only that, for a system, a connection exists between x and y. This connection may arise from temporal proximity, similarity, repeated co-occurrence, organismic relevance, or a learned action coupling.

The transition through the four dimensions can thus be represented schematically as follows:

$$D1: \quad x \sim y$$

$$D2: \quad R(x, y)$$

$$D3: \quad M\big(R(x, y)\big)$$

$$D4: \quad M(R(x, y) \mid C)$$

D1 establishes a connection. D2 specifies the character of this connection as a relation R. D3 makes the relation itself, together with its presuppositions, an object of reflection. D4 additionally specifies the context C within which the relation is formulated and tested, while simultaneously marking the choice of this context as perspectival.

This representation does not imply a rigid temporal sequence. Human beings continually move between the dimensions. A mathematical idea may arise intuitively at D1, be formally tested at D2, examined with respect to its axioms at D3, and situated within a particular theoretical context at D4. Conversely, a D3 reflection may trigger a new D1 association. The dimensions are therefore not developmental stages that disappear once passed through, but epistemic modes of operation that remain simultaneously available.

2. D1: Associative Linking

At D1, patterns are connected immediately. A classical example is:

tiger ~ danger ~ flight

No explicit theory of the tiger is required for this connection. An organism need neither define the concept of danger nor perform a causal analysis. It is sufficient that a perceived pattern is coupled with a state of high relevance and with an action. Similar connections include

fire ~ heat ~ distance

or

smell of food ~ approach

The associative characterization of D1 does not replace the binary distinction already presupposed. Rather, it presupposes it. Only states that are treated by a system as different can be connected with one another. The elementary distinction $x \neq y$ and the subsequent connection $x \sim y$ are therefore not competing operations. The distinction supplies the difference; association couples the differentiated states. The basic function of bivalent differentiation is thereby preserved.

D1 should therefore not be equated with primitive irrationality. On the contrary, immediate pattern connections are indispensable for a living organism. An organism that had to perform a complete analysis before every response would be incapable of acting in many situations. The speed of associative processing is a substantial biological advantage.

This strength, however, has an epistemic reverse side. D1 does not necessarily distinguish among temporal succession, similarity, correlation, functional dependence, and causation. Two events may be firmly connected for a system even though the nature of their relation remains undetermined. In everyday life this is often sufficient. In science and philosophy it is not.

D1 therefore initially generates possible connections. It supplies candidates for further determination. The connection

$$x \sim y$$

is epistemically weak but productive. It may be false, accidental, or irrelevant. It may also form the starting point of a scientific discovery. Many theoretical insights begin with the recognition of a similarity or a recurring pattern. The error therefore does not lie in association itself. It arises only when the epistemic status of an association is altered without being noticed.

3. D2: Rule-Guided Relation Formation

D2 begins where an open connection is formulated as a determinate relation. From

$$x \sim y$$

we obtain

$$R(x, y)$$

Here R may have very different meanings. It may express equality, similarity, membership, logical implication, mathematical dependence, statistical correlation, functional coupling, or causation. D2 therefore forces a determination of what, at D1, was merely connected.

In classical logic, for example, we may have

$$A \rightarrow B$$

and

$$A$$

therefore

$$B$$

The validity of this inference does not depend on whether a person psychologically associates A with B. It follows from the rule of inference specified within the system. Likewise, a mathematical relation derives its validity from definitions, axioms, and admissible operations.

An important distinction thus becomes visible. The discovery of a relation may occur associatively, but its justification need not. A mathematician may suddenly recognize that an unfamiliar problem resembles a familiar structure. This intuitive connection initially belongs to D1. Whether the presumed structure is actually present must then be tested at D2.

This can be represented as a transition:

$$x \sim y \quad \Rightarrow \quad R(x, y)$$

The symbol ⇒ here does not denote logical implication, but an epistemic transition from an initially open connection to a determinate relation.

This transition creates a characteristic risk of error. Co-occurrence, in particular, is easily transformed into a causal relation:

$$A \sim B \quad \rightsquigarrow \quad A \rightarrow B$$

The original observation may be correct while the causal interpretation is false. The fact that A regularly occurs with B warrants investigation. It does not prove that A causes B.

D2 is therefore more powerful than D1, but not automatically closer to truth. An incorrectly specified relation can be processed with great formal consistency. Formal correctness does not protect against mistyped premises. This is precisely where D3 becomes necessary.

4. D3: Reflexive Examination

D3 does not merely introduce additional objects of inquiry. Rather, the operation is directed toward the structure of the relation previously formed. The questions now concern which presuppositions are contained in $R(x, y)$, which categories are employed, within which language x and y are defined, and what kind of relation R can be in the first place.

Formally, the transition is:

$$R(x, y) \quad \rightarrow \quad M\big(R(x, y)\big)$$

Here M denotes a reflexive operation. It examines the relation not only with respect to its result, but with respect to its conditions.

In mathematics, D2 can conduct a proof within an axiomatic system. D3 asks what role the axioms employed play, whether a theorem is independent of them, which alternative axiomatic systems are possible, or which properties are already presupposed by the chosen formalism. In this sense, D3 is metatheoretical.

In the empirical sciences, D3 also concerns the question of whether the terms used in a proposition belong to the same level of description. Here an extension of the existing dimensional logic becomes possible. Levels of description are not identical with dimensions of thought. D1 to D4 specify how epistemic operations are performed. Levels of description specify the language or methodological access through which an object is described.

An organism, for example, may be described physiologically, psychologically, and phenomenologically. These three descriptions do not produce three organisms or three modes of being. They refer to the same process under different conceptual and methodological conditions.

D3 is the dimension in which such conditions can be made explicit.

5. Levels of Description and Dimensions of Thought

The distinction between levels of description and dimensions of thought is crucial. Equating the two would produce precisely the ontologization that dimensional logic is intended to avoid.

Let O be an object or process. Different systems of description can generate different representations of the same object:

$$D_{L_1}(O) = x_1$$

$$D_{L_2}(O) = x_2$$

$$D_{L_3}(O) = x_3$$

Here L_i denotes a level of description and D_{L_i} the corresponding description. The expressions x_1, x_2, and x_3 need not be mutually translatable. Nevertheless, they refer to the same object O.

For a human being, for example, we may have:

$$D_{phys}(O) = \text{neural state}$$

$$D_{psych}(O) = \text{decision behavior}$$

$$D_{phen}(O) = \text{experienced decision}$$

The three expressions are not three parts of the human being. They belong to three ways of describing the same organismic process.

Within each level of description, all four dimensions of thought can occur. A physiological hypothesis may arise at D1 from an observed connection, be formulated at D2 as a causal model, be analyzed at D3 with respect to its presuppositions, and be restricted at D4 to its experimental domain of validity. The same is possible for psychological or phenomenological propositions.

This yields a cross-structure. Dimension of thought and level of description are independent axes. One axis concerns the epistemic operation; the other concerns the language in which the object is framed.

6. The Level Operator

To capture mixtures of levels more precisely, a level operator Λ is introduced. It assigns each term to the level of description within which that term is defined:

$$\Lambda(x) = L_i$$

For a neural state N, for example:

$$\Lambda(N) = L_{phys}$$

For a pain experience S:

$$\Lambda(S) = L_{phen}$$

For a reported decision E:

$$\Lambda(E) = L_{psych}$$

The level operator says nothing about what the object is ontologically. It classifies only the expression within a system of description. The operation therefore remains epistemic.

Let $\mathcal{L}$ be the set of all levels of description. In addition, each relation receives a signature. This signature is a set of admissible pairs of levels and specifies between which levels of description the relation is defined. For a physiological causal relation C_{phys}, for example, we may have:

$$\sigma(C_{phys}) = \{(L_{phys}, L_{phys})\}$$

The proposition

$$C_{phys}(N_1, N_2)$$

is then well-defined if

$$\Lambda(N_1) = \Lambda(N_2) = L_{phys}$$

A phenomenological relation R_{phen} could correspondingly have the signature:

$$\sigma(R_{phen}) = \{(L_{phen}, L_{phen})\}$$

A simple well-formedness condition can now be formulated. For a binary relation R with signature

$$\sigma(R) \subseteq \mathcal{L} \times \mathcal{L}$$

the expression

$$R(x, y)$$

is type-correct only if

$$\Lambda(x) = L_i$$

and

$$\Lambda(y) = L_j$$

The general condition is:

$$WF(R(x, y)) = 1 \Leftrightarrow (\Lambda(x), \Lambda(y)) \in \sigma(R)$$

WF denotes well-formedness.

This typing explicitly does not prohibit relations between different levels of description. An empirical correlation between a neural state and a reported experience can be defined precisely as a cross-level relation:

$$\sigma(K_{phys,phen}) = \{(L_{phys}, L_{phen})\}$$

Then

$$K_{phys,phen}(N, S)$$

is entirely admissible.

The decisive issue is therefore not that different levels occur in a proposition. The decisive issue is whether the relation employed is defined for this transition between levels.

7. Mixing Levels as a Type Error

The level operator allows a category error to be specified more precisely. Consider again a physiological causal relation:

$$\sigma(C_{phys}) = \{(L_{phys}, L_{phys})\}$$

and the terms

$$\Lambda(N) = L_{phys}$$

and

$$\Lambda(S) = L_{phen}$$

Then the expression

$$C_{phys}(N, S)$$

is not type-correct, because

$$(\Lambda(N), \Lambda(S)) = (L_{phys}, L_{phen}) \notin \sigma(C_{phys})$$

thus

$$WF\left(C_{phys}(N, S)\right) = 0$$

The error is not that neural states and experience have nothing to do with one another. The error lies in applying a relation defined within a physiological description to a phenomenological term without additional justification.

The concept of a category error can therefore be sharpened:

$$WF(R(x, y)) = 0 \Rightarrow \text{unjustified relation transfer}$$

This condition is initially syntactic and epistemic. A value of $WF = 0$ shows that the expression is not well-defined relative to the specified typing. By itself, this does not yet prove a philosophical category error, because both the assignment of levels and the signature of the relation themselves require justification at D3. A category error is present when the type violation cannot be resolved by a justified cross-level relation, transformation, or redefinition of the relation.

Let a transformation between two levels of description be given by

$$T_{ij}: L_i \to L_j$$

A relation within L_i does not, without further argument, license the same relation within L_j. In particular, the following does not hold in general:

$$R_i(x_i, y_i) \nRightarrow R_j\left(T_{ij}(x_i), T_{ij}(y_i)\right)$$

A sufficient preservation condition would be:

$$\forall x_i, y_i: \quad R_i(x_i, y_i) \Rightarrow R_j\left(T_{ij}(x_i), T_{ij}(y_i)\right)$$

Only if such structural compatibility has been demonstrated may the relevant relation be treated as preserved across the transition. Stronger claims of structural identity would require a corresponding equivalence condition.

For many scientific levels of description, precisely this condition is not given. A physiological concept does not simply become the same concept in different notation when transferred into a psychological description. The description changes its categories.

8. The Unmarked Transfer of a Relation

The typical level error can now be described as an unmarked relation transfer. The starting point is a relation R_i that is meaningfully defined at a level of description L_i. The same relational form is then transferred to a term belonging to a different level L_j without specifying a cross-level signature or transformation.

Formally:

$$R_i(x_i, y_i)$$

becomes

$$R_i(x_i, y_j)$$

even though

$$\Lambda(y_j) = L_j \neq L_i$$

and

$$\sigma(R_i) = \{(L_i, L_i)\}$$

The transfer is therefore not justified.

Natural language easily conceals transitions of this kind. Verbs such as “cause,” “decide,” “know,” “store,” or “represent” are used across different descriptive systems without marking the shift in meaning. This creates the impression that the same relation is merely being applied to a different object. In fact, however, the semantic and methodological signature of the concept may have changed.

D3 therefore has more than the task of reflecting on axioms or theories. D3 also controls the typing of the terms and relations employed. In this respect, the reflexive dimension functions as epistemic error control.

9. Example: The Brain Decides

The statement “the brain decides” illustrates the problem particularly clearly.

Empirically, it can be established that certain neural changes occur before an action. Within a physiological description, causal relations among neural states can be investigated:

$$C_{phys}(N_1, N_2)$$

with

$$\sigma(C_{phys}) = \{(L_{phys}, L_{phys})\}$$

"Deciding," by contrast, is in ordinary psychological language a description of the behavior of an organism or person. Let E be a corresponding term:

$$\Lambda(E) = L_{psych}$$

If we now formulate

$$C_{phys}(N, E)$$

then, without further definition, a type shift has occurred. The proposition does not count as a physiological causal claim merely because a neural state temporally precedes a reported decision.

This does not mean that decisions occur without neural processes. As metonymic shorthand, the sentence "the brain decides" may be entirely unproblematic. It becomes a category error only when this shorthand is used to derive an explanatory separation of brain and person as two competing agents. "Neural process" and "decision" may be different descriptions of the same organismic event. Turning this difference into a competition between two acting instances creates the apparent problem of whether the brain or the person decides.

D3 does not solve this problem by introducing a new empirical theory. It shows that the question itself may already be mistyped.

10. Example: Genes Determine Intelligence

A similar error occurs in statements about genes and intelligence. Genetic variants may correlate with differences in particular cognitive performances. Likewise, altering a gene may causally alter biological processes. It does not follow, however, that intelligence as a system property is located in a gene.

Let G be a genetic state with

$$\Lambda(G) = L_{mol}$$

and let I be a description of organismic intelligence with

$$\Lambda(I) = L_{org}$$

A molecular-biological causal relation may have the signature:

$$\sigma(C_{mol}) = \{(L_{mol}, L_{mol})\}$$

or, extended to immediate cellular consequences,

$$\sigma(C_{mol,cell}) = \{(L_{mol}, L_{cell})\}$$

The global statement

$$G \to I$$

counts as an explanation only if the transition from molecular changes through development, cellular organization, the nervous system, learning, and environmental coupling is made explicit. The manipulability of one factor does not establish an explanatory reduction of the whole system to that factor.

This also shows that level errors are connected with a widespread error in causal reasoning. Local intervention efficacy is transformed into global explanatory power. A component may be necessary for the stability of a system without by itself explaining the system property.

The level operator makes visible that "gene" and "intelligence" already belong to different descriptive types. D3 therefore forces the transition to be justified instead of allowing it to be skipped linguistically.

11. Functional Equivalence and the Transfer of Consciousness

Functionalism provides a particularly suitable test case for dimensional error analysis. A human being and a machine can perform the same task with respect to a clearly specified function. A human H and a machine M, for example, may correctly perform the same calculation. With F_R as the functional description of calculation, we may have:

$$F_R(H) = F_R(M)$$

This equality may be empirically entirely correct. It can be expressed as functional equivalence with respect to the relevant calculating function:

$$H \equiv_{F_R} M$$

The relation is indexed, however. It states only that H and M are equivalent with respect to F_R. It does not state that the two systems possess the same properties in every other respect. For a human being, there may additionally be a phenomenal property E:

$$E(H)$$

Without an additional premise, functional equivalence does not entail phenomenal equivalence:

$$H \equiv_{F_R} M \nRightarrow H \equiv_E M$$

The difference can be made explicit using the level operator. The proposition about calculating performance belongs to a functional description, while the proposition about experience belongs to a phenomenal description:

$$\Lambda\big(F_R(H)\big) = L_{func}$$

$$\Lambda\big(E(H)\big) = L_{phen}$$

This does not rule out a relation between the two levels. Such a relation must, however, be stated explicitly. Anyone who wishes to infer experience E from a functional organization F_C requires at least a bridge premise of the form:

$$\forall x \, [F_C(x) \Rightarrow E(x)]$$

The simple error of inference therefore consists in turning partial functional equivalence into property equivalence simpliciter. From the fact that a human and a machine both calculate, it does not follow that every further property of the human can be transferred to the machine. D3 makes visible that an additional relation must be introduced whose validity requires independent justification.

A strong functionalist can avoid this simple objection by defining consciousness itself functionally. The inference is then no longer from calculation to consciousness. Rather, the claim is that a certain functional organization F_C is sufficient for experience or identical with it. The bridge premise already introduced is a sufficiency claim. An identity claim is stronger. In its strongest form it is:

$$\forall x\,[E(x) \Leftrightarrow F_C(x)]$$

The simple type error now initially disappears because the functional and phenomenal properties are linked by an additional identity assumption. The burden of justification, however, shifts to D3. It must now be shown why precisely this functional organization should be sufficient for experience or identical with it. The co-occurrence of F_C and E in biological organisms underdetermines this choice: by itself it establishes neither universal sufficiency nor identity nor substrate independence. This is not a refutation of functionalism. Rather, dimensional logic locates the additional premise on which the force of the functionalist inference depends.

D4 adds a second control. Functional equivalence always holds with respect to a particular function and within a particular context C:

$$H \equiv_{F_R,C} M$$

If this index is omitted, a local equality can be transformed into an equality of systems. This transition is not valid:

$$H \equiv_{F_R,C} M \nRightarrow H \equiv M$$

The functionalism example thus reveals two distinct kinds of error. The first is an illegitimate transfer of properties from functional to phenomenal equivalence. The second is not a formal mixing of levels, but an independently unjustified assumption of sufficiency or identity. D3 examines its categorial and theoretical legitimacy. D4 prevents an equivalence restricted to a function and a context from being treated as equivalence of the entire systems.

12. Example: Consciousness

Mixing levels is particularly consequential in consciousness research. Robust correlations are found between neural states and reported or otherwise inferred conscious states. This can be represented formally as a cross-level empirical relation:

$$K_{phys,phen}(N, E)$$

with

$$\sigma(K_{phys,phen}) = \{(L_{phys}, L_{phen})\}$$

Such a correlation is type-correct because its signature explicitly provides for a comparison between two levels of description.

The transition to

$$C_{phys}(N, E)$$

becomes problematic if C_{phys} continues to be understood as a physiological causal relation. A phenomenological term is then inserted into a relation that is defined for physiological terms.

Thus, from

$$N \sim E$$

we initially obtain

$$K_{phys,phen}(N, E)$$

and eventually, without the transition being noticed,

$$C_{phys}(N, E)$$

The first step may be empirically legitimate. The second requires additional theoretical justification.

At precisely this point, the linguistic formulation “the brain generates experience” can suggest an ontological duality that has itself been produced by the scientific description. First, an organismic state is described physiologically. The same or a corresponding state is described phenomenologically. The two descriptions are then treated as if they were two separate processes and causally connected with one another.

The alternative is not to identify physiology and phenomenology. It is to type their relation correctly. For an organism O, we may have:

$$D_{phys}(O) = N$$

and

$$D_{phen}(O) = E$$

Then N and E are two descriptions of the organism under different epistemic modes of access. It does not automatically follow that

$$N \rightarrow E$$

Nor does it follow that the two descriptions have nothing to do with one another. The task of a theory is to reconstruct the constitutive organization of the organism and connect the different descriptions to it without reinterpreting them as separate domains of being.

13. D4: Contextual and Perspectival Validity

D3 can establish that a relation is type-correct. This does not yet determine how far the relation is valid. That question belongs to D4. D4 does not merely add another boundary condition. In the revised transition logic, the standpoint from which a context is selected and validity is determined is itself marked as perspectival. No context thereby acquires the status of a final or neutral framework.

For the error-control problem examined here, this contextual binding can be represented in simplified form not merely as

$$R(x, y)$$

but as

$$R(x, y \mid C)$$

C denotes a context that specifies the conditions under which the relation is valid. This notation is an abbreviation and should not be equated with the full D4 operation. Context may include experimental conditions, theoretical frameworks, historical situations, social institutions, biological developmental states, or defined mathematical structures.

D4 therefore does not make propositions arbitrarily relative. On the contrary, specifying a context increases precision. A proposition that is valid only under certain boundary conditions does not become more true when those conditions are omitted. It merely becomes more general than its justification warrants.

A contextualized well-formedness condition can therefore be added:

$$WF_C\big(R(x, y)\big) = 1$$

when both the typing and the contextual conditions are satisfied.

This can be written as a conjunction:

$$WF_C\big(R(x, y)\big) = WF\big(R(x, y)\big) \wedge Valid(R, C)$$

D4 thereby becomes a test of the scope of a relation that is already categorially admissible.

A causal relation, for example, may occur stably in an experimental setting and be overlaid by additional variables outside that setting. A moral rule may be unambiguous within one institutional framework and applied differently in another context. A mathematical theorem may hold within one axiomatic system and fail to be derivable within another.

D4 therefore does not simply ask: Is the proposition true? D4 asks: For which explicitly specified context is its truth being claimed?

14. From Association to Category Error

With the four dimensions and the level operator, a general error path can now be reconstructed.

At the beginning is a D1 connection:

$$A \sim B$$

At D2 it becomes a determinate relation:

$$R(A, B)$$

In many cases this relation is further strengthened:

$$A \rightarrow B$$

D3 would now have to test whether the terms are type-correct:

$$WF\big(R(A, B)\big) = 1?$$

D4 would additionally have to ask whether the relation holds in the claimed context:

$$WF_C\big(R(A, B)\big) = 1?$$

If D3 and D4 are skipped, a dimensionally truncated inference results:

$$D1 \rightarrow D2$$

without the control operations

$$D3 \rightarrow D4$$

This truncated path explains why many problematic statements appear immediately plausible. The original association may be empirically well grounded. Precisely for that reason, its later upgrading is easily overlooked.

“Dopamine makes us happy,” “genes determine intelligence,” “the brain decides,” “information generates consciousness,” or “the model represents reality” possess strong linguistic plausibility. They compress complex relations into simple cause-and-effect schemes. D3 shows, however, that different descriptive types are often connected with one another. D4 additionally shows that the relevant domain of validity often remains concealed.

The formula for this error is therefore not simply “false association.” More accurately:

$$\text{association} \neq \text{relation} \neq \text{explanation}$$

An association is a possible connection. A relation specifies the kind of connection. An explanation must additionally show why this relation is categorially admissible and within which context it is valid.

15. Four-Dimensional Logic as Epistemic Error Control

With this extension, dimensional logic acquires an additional function. It does not merely describe different forms of thinking. It can also be read as a procedure of epistemic error control.

D1 answers the question of why two contents are connected at all. D2 asks which determinate relation is being asserted between them. D3 examines the categories, levels of description, axioms, and metatheoretical presuppositions of this relation. D4 specifies the context of validity and simultaneously marks that this context itself is not selected from a neutral standpoint.

This stratification does not replace domain-specific methods. Statistical causal analysis, mathematical proof, experimental control, and semantic analysis remain necessary. Dimensional logic instead orders the point at which each kind of examination becomes necessary.

Of particular importance is the distinction between formal correctness and epistemic adequacy. An inference can be entirely correct within a formal system while nevertheless proceeding from unsuitable premises. If the premises already contain unmarked shifts between levels, a formally correct inference merely reproduces the original category error.

D3 therefore provides a necessary metatheoretical control of formal operations. D4 additionally prevents locally valid propositions from appearing as universal claims or a particular framework of validity from being tacitly treated as privileged.

16. Consequences for the Concept of Intelligence

The extension also has consequences for the concept of intelligence. Intelligence can no longer simply be identified with logical inferential ability or rapid pattern recognition. Both capacities concern only parts of the dimensional spectrum.

A system may perform extraordinarily well at D1 by recognizing patterns and selecting successful responses. At D2 it may apply complex formal rules. Nevertheless, it can still produce errors if it is unable, at D3, to examine the presuppositions of its own relation formation or, at D4, to take their contextual dependence into account.

From this perspective, intelligence would not be maximal performance within a single dimension. As a heuristic consequence of dimensional logic, it can instead be understood as the capacity to move appropriately among the dimensions and activate the operation required by a given task. This characterization is explicitly conceptual and is not intended as a quantitative definition.

An intelligent system would not have to operate constantly at D4. That would be inefficient. It would, however, have to recognize when an immediate association is sufficient, when rule-governed testing is necessary, when a relation must be reflexively examined with respect to its presuppositions, and when context is decisive for its validity.

This also gives associative thinking a more precise status. Association is not an inferior preliminary stage of rational thought. It is the generative basis from which possible connections emerge. Rationality arises not by abolishing association, but by subjecting it to dimensional control.

17. Conclusion

Four-dimensional logic can be further developed as a theory of epistemic relation formation while preserving its basic structure. D1 denotes the immediate and reactive dimension, D2 the systematic and rule-guided dimension, D3 the reflexive dimension, and D4 the contextual and perspectival dimension in which the chosen standpoint itself is also relativized. What is new is the explicit question of what kind of connection is established and examined in each of these dimensions.

D1 generates associations. D2 specifies them as relations. D3 examines the categories and presuppositions of these relations. D4 determines their contextual validity and marks the perspectival character of the chosen framework of validity.

The level operator Λ supplements this structure with a formal typing. Each term is assigned to a level of description. Each relation receives a signature as a set of admissible pairs of levels. This makes it possible to test whether a proposition is well-defined within the categories it employs. Relations between different levels of description remain possible provided that they are explicitly defined as such. What is prohibited is not a transition between levels, but an unmarked transition.

A widespread category error can therefore be described formally. A relation whose signature admits certain pairs of levels is applied to a term belonging to another level without specifying a bridge relation or transformation. Formally, this initially reveals a type violation. Whether the type violation is to be judged a category error depends on whether a cross-level relation or transformation can be justified.

The extension also makes understandable why such errors are so common. They often begin with an entirely justified D1 association. Two events co-occur, an intervention changes an outcome, or a neural state correlates with an experience. Only in the transition to D2 does the connection become a stronger relation. If D3 and D4 are omitted, a plausible observation becomes a categorially inadmissible or contextually overextended explanation.

The functionalism example additionally shows that the same control applies to claims of equivalence. Partial equality with respect to a function must neither be extended into equality of the systems nor transferred to phenomenal properties without additional justification.

The central formula is therefore:

$$\boxed{\text{association} \neq \text{relation} \neq \text{explanation}}$$

Four-dimensional logic thus describes not only how thinking generates relations. It also shows how thought can control its own relations. This is precisely its epistemic function.

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